Revision Lecture

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L1: Mathematical Methods I

- Optimisation problem of the form

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to

$$c_t + k_{t+1} - (1-\delta)k_t = w_t + r_t k_t$$

has Lagrangian of the form

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} + \sum_{t=0}^{\infty} \lambda_t [w_t + r_t k_t - c_t - k_{t+1} + (1-\delta)k_t]$$

L1: Mathematical Methods I

- Solve it by taking derivatives with respect to the control variables.
- Set these derivatives equal to zero to get first order conditions

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0$$
$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = 0,$$

  which, together with the budget constraint, summarise the solution to the system.
- If there is randomness in the model, then we put expectations around the Lagrangian. If not, no need.
- Log-linearisation: $x_t = \bar{x}e^{\hat{x}_t}$ where $\hat{x}_t = \log(x_t) - \log(\bar{x})$.
- Always collect the exponential terms together as best you can before invoking the Taylor expansion $e^{\hat{x}_t} \approx (1 + \hat{x}_t)$.

L2: Real Business Cycle Model

- Stochastic model with productivity shocks.
- Key equation

$$\log(a_t) = \rho_a \log(a_{t-1}) + \epsilon_{a,t}, \quad \epsilon_{a,t} \sim N(0, \sigma^2)$$

  where $0 < \rho_a < 1$.
- Realisations to the shock term $\epsilon_{a,t}$ are what drive business cycles.
- Size of the persistence term ρ_a determines how long it takes to return to steady state.
- Competitive equilibrium yields same solution as social planner's problem.
- Business cycles are a natural part of life.
- No money; need a numeraire good.

L3: Old School Theories and Modelling Money

- IS-LM.
- Know what all the curves mean. A good way to test your understanding is to look at the ZLB lecture exercise set.
- Just stick money into the RBC model without frictions: creates problems.
- Won't get any demand for money as it will be dominated by other assets, which offer a positive return.

L4: Money in the Utility Function Model

- First attempt at modelling money.
- Augment preferences with a desire for real balances

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\varphi}}{1+\varphi} + \frac{(m_{t+1}/p_t)^{1-\nu}}{1-\nu} \right]$$

  which generates a demand curve for money.
- Other key equation: central bank's money supply.
- Money demand and supply determine equilibrium in money market.
- This approach is just like sticking another good into the model.
- With separable utility across consumption, labour and real balances, the money market equilibrium has no impact on other real variables.

L5: Cash in Advance Model

- Generates money demand by assuming certain types of goods require cash for purchase.
- Other goods not requiring cash are referred to as credit goods.
- The effect of money on the economy depends strongly on which goods are assumed to require cash.
- Arbitrary: this is no good.
- E.g. if consumption goods require cash, then you'll have constraint

$$p_t c_t \leq m_t + \tau_t$$

where τ_t is a government transfer of cash to the household (may be zero or non-zero).

L6: Overlapping Generations Model

- Assumes that agents only live for a certain number of periods.
- New generation born at each time t.
- The equilibrium of the model can be inefficient.
- Nobody wants to lend money to the old since they can't pay it back.
- Creates a role for money: old can give money to the young.
- Only works in a monetary equilibrium (when money is valued).
- Money will be valued when people today think it will be valued tomorrow.

L7: New Keynesian Part I: Imperfect Competition

- First ingredient into the NK model.
- Introduced a static model with monopolistically competitive firms: each makes a different variety $j \in [0, 1]$.
- Households have preference over varieties $j \in [0, 1]$.
- Households undertake two-stage budgeting:
  - **(1)** How much of each variety do I want to consume?
  - **(2)** Given my answer to (1) above, how much do I want to consume in total?
- Step (1) generates demand curves $C(j) = \left(\frac{P(j)}{P}\right)^{-\epsilon}$.
- Step (2) generates the labour supply condition.

L7: New Keynesian Part I: Imperfect Competition

- Firms maximise their profits given the answer to (1) above, i.e. $C(j)$.
- Optimal solution is to set price as a markup over marginal cost.
- Markup depends on the elasticity of substitution across different varieties.

L8: New Keynesian Part II: Price Stickiness

- Second ingredient into the NK model.
- Calvo price setting: θ is the probability that a firm will be stuck with the price they set today tomorrow.
- Optimal price when we get a re-set (probability $1-\theta$) maximises the expected value of future profits given this price we choose today.
- Discount future profits using the household's stochastic discount factor.
- Household's stochastic discount factor is generally different from β. Why?

L8: New Keynesian Part II: Price Stickiness

- Rotemberg price setting allows prices to change at an adjustment cost.
- Difference between the two approaches: price dispersion with Calvo and none with Rotemberg.
- No dispersion with Rotemberg since all the firms are the same.

L8: New Keynesian Part III: NK Phillips Curve

- Old Keynesian Phillips curve: empirical relationship between the state of the economy and inflation: lower unemployment can only happen with high inflation.
- Old idea criticised based on a lack of micro-foundations and failure to account for future expectations.
- New Keynesian Phillips curve incorporates these features

$$\hat{\pi}_t = \kappa \hat{y}_t^g + \beta \mathbb{E}_t[\hat{\pi}_{t+1}]$$

where κ parameter gets at the old Phillips curve idea and the expectation accounts for the future inflation.

L10: Solving DSGEs Part I: Analytical Methods

- Method of undetermined coefficients or "guess and verify".
- Always guess that the endogenous variables are functions of the state variables and shocks.
- Recall that the state variables summarise the "state" of the system at time t. They contain all the necessary information.
- E.g. in RBC model, there are productivity shocks and last period's capital stock and productivity is the state variable. So you'd conjecture, say for next period's capital, that

$$\hat{k}_{t+1} = \phi_a \hat{a}_{t-1} + \phi_k \hat{k}_t + \phi_\epsilon \epsilon_{a,t} \tag{1}$$

where the ϕ terms are coefficients that we need to find.

L10: Solving DSGEs Part I: Analytical Methods

- This method says, we'll make an assumption about the form of the solution like in equation (1) (the guess part).
- We'll then substitute these guesses into the system of equations.
- Then gives us restrictions on the coefficients ϕ such that the guess indeed is the solution (the verify part).
- We use this method if the model is simple enough. Will give us the parameters we in (1) as functions of the parameters of the problem (e.g. utility function parameters etc).

L11: Solving DSGEs Part II: Numerical Methods

- We turn to numerical solutions if the model is too complicated for analytical methods.
- Can use Dynare easily for this purpose.
- Tells us what the policy functions are, (i.e. equation (1) is the policy function for capital's choice), for specific parameter values (e.g. $\sigma = 2.0$).
- We use these solution techniques for answering quantitative questions, (e.g. if monetary policy increases by 1%, how much will output decrease by?).

L12: New Keynesian Part IV: Optimal Monetary Policy

- Looked at optimal policy in the Calvo context.
- The deadweight losses of Calvo come from the price dispersion.
- Higher inflation means more price dispersion.
- Optimal policy was to achieve the flexible price equilibrium.
- No price dispersion in this case.
- Optimal policy was $\hat{i}_t = \hat{r}_t^n$ in equilibrium.
- Taylor principle: we need the central bank to target inflation to prevent a hyperinflation.

L13: New Keynesian Part V: Fiscal Multipliers

- The multiplier studies the effectiveness of fiscal policy on the macroeconomy.
- Percentage change in GDP coming from a 1% increase in government spending.
- The multiplier is larger with sticky prices than with flexible prices.
- Why? These models always assume that the goods market clears: never any shortages.
- Output needs to adjust more to clear the goods market than it has to with perfect price adjustment.

L14: New Keynesian Part VI: Zero Lower Bound

- We're at the zero lower bound when the net nominal interest rate equals zero.
- Happens when the economy is in bad shape.
- The fiscal multiplier is larger at the ZLB.
- Why? Less "crowding-out" since the nominal interest rate is non-responsive to the fiscal shock.

L15: Mathematical Methods II

- Dynamic programming approach to discrete time problems. We can write the recursive formulation to the problem

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to

$$c_t + k_{t+1} - (1-\delta)k_t = w_t + r_t k_t$$

as

$$V(k) = \max_{c, k'} \frac{c^{1-\sigma}}{1-\sigma} + \beta V(k')$$

where $V(k)$ is the value function associated with state k.

- The solution to the recursive formulation is policy functions $c(k)$ and $k'(k)$: optimal choices of controls as a function of the state.

L15: Mathematical Methods II

- Recursive formulation solution is policy functions $c(k)$ and $k'(k)$ as opposed to the sequence problem solution $\{c_t, k_{t+1}\}_{t=0}^{\infty}$, which is an infinite sequence of optimal controls.

- Continuous time value functions are of the form discount rate times value of state equals probability times flow payoff plus change in value function.

L16: New Monetarism Part I: First Generation Models

- Assume 1 unit of money and 1 unit of goods.
- Kiyotaki and Wright (1993).
- Micro-foundations: money as a medium of exchange.
- When the probability of a double-coincidence is sufficiently small then money can serve a role by facilitating exchanges.
- Optimal amount of money in circulation depends on how large the probability of a double coincidence. If it's really high, then don't bother with money as it squeezes-out the possibility of production for some agents.

L17: New Monetarism Part II: Second Generation Models

- Trejos and Wright (1995).
- One unit of money and divisible goods.
- Creates room for prices.
- Agents meet each other randomly and then negotiate over how many goods will be exchanged for a unit of money (reciprocal of the price).
- Bargaining solution maximises the Nash product subject to constraints.
- Nash bargaining all about splitting the surplus.

L18: New Monetarism Part III: Third Generation Models

- Lagos and Wright (2005).
- Divisible money and divisible goods.
- Day (decentralised) and night (centralised) market setup with quasi-linear preferences required for analytical tractability.
- We get a degenerate distribution of money holdings at the start of each period, the night market re-sets people's future benefit to holding cash.
- Doesn't matter what happened in the day market beforehand.

L19: Finance Part I: Asset Pricing

- Lucas (1978) "tree" model.
- Household chooses how much of a risky asset to hold, a_{t+1}. The budget constraint would look like

$$p_t a_{t+1} + c_t \leq (p_t + d_t) a_t$$

  if the risky asset were the only one in the economy. That is: your holdings of the stock from last period a_t yield you a divined d_t and they're worth p_t. You sell these shares and re-optimise this period by spending $p_t a_{t+1}$.
- In equilibrium, $a_{t+1} = 1$ if the shares are in unit net supply (total shares sum up to one).
- Equity premium puzzle: return of risky assets over riskless in the data implies incredible high risk aversion coefficient for the household. Something's wrong with the model!

L20: Finance Part II: Corporate Finance

- A firm wants to take on a project. What's the best way to finance it?
- Modigliani and Miller (1958): if there are no "financial frictions" then it doesn't matter how you finance it. Debt and equity are equivalent.
- Only holds under the following conditions:
  - Perfect and complete capital markets,
  - No taxes,
  - Bankruptcy is not costly,
  - Capital structure won't affect investment decisions,
  - Symmetric information between insiders and outsiders.
- Depending on which assumptions you relax, equity may be better or worse than debt! Can create value for the owners.