Lecture 7: New Keynesian Model Part I
Imperfect Competition

Adam Hal Spencer

The University of Nottingham

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Roadmap

Motivation

- New Keynesian model: the "standard" DSGE framework of the modern era.
- Used by central banks all over the world to study the impact of monetary (and fiscal) policies on the macroeconomy.
- Nice because we can use a computational software called *Dynare*, (to be discussed later).
- Essentially the RBC model with imperfect competition, sticky prices and monetary policy.
- Gali (2008), "Monetary Policy, Inflation and the Business Cycle" is a good beginner's-guide to these types of models.
- Woodford (2003), "Interest and Prices" is a little more advanced; basically the bible for DSGEs.

This Lecture

- We'll look at the first ingredient of the model: the departure from perfectly competitive firms.
- Build a model of firms with differentiated products in a static general equilibrium environment.
- Krugman (1980), "Scale Economies, Product Differentiation and the Pattern of Trade", *American Economic Review*.

Roadmap

Setup

- Model of monopolistic competition.
- In the RBC model, we had that firms were price takers and perfectly competitive.
  - Firms would make zero profits (price equal to marginal cost).

Setup

- Assume that firms make different varieties of goods.
- Given that goods aren't perfect substitutes, the firms will generate positive profits with price above marginal cost.
- Firms are sufficiently small though that they won't have an impact on overall industry aggregates.
- Continuum of varieties over the interval $[0, 1]$.
- Firms use labour as sole factor of production.

Setup

- Households have a love of variety across the continuum.
- Supply labour to the firms for a wage.

Household Problem

- Household consumption is an aggregate across all the different varieties

$$C = \left(\int_0^1 C(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 1$ is the elasticity of substitution across varieties.

- Notice that when $\epsilon \to \infty$, the goods become perfect substitutes as

$$C = \left(\int_0^1 C(j) dj \right)$$

meaning that it doesn't matter which good is consumed.

- In principle, firms can also have complementary products if $\epsilon < 1$.

Household Problem

- Budget constraint across the continuum of varieties

$$\int_0^1 P(j)C(j)dj \leq WN$$

where W is the wage and N is their labour supply.

- Utility across consumption and leisure

$$U(C,L) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$$

Household Problem

- Two-stage budgeting program.
  (1) Allocate spending across varieties for a given level of income.
  (2) Choose between consumption and leisure.

Household Problem: Stage (1)

- How much of each variety should I consume?
- Household solves the program

$$\max_{\{C(j)\}_{j\in[0,1]}} C = \left(\int_0^1 C(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$$

subject to

$$\int_0^1 P(j)C(j)dj \leq WN$$

which yields optimal demand curve for each variety $C(j)$ for $j \in [0,1]$.

- Then gives us the objective — aggregate consumption — to feed into the next stage of optimisation.

Household Problem: Stage (2)

- Maximise utility through the tradeoff between consumption and leisure

$$\max_{\{C,N\}} \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$$

subject to

$$PC \leq WN$$

where P is some price index across the varieties, (depends on the solution at stage 1, to be discussed later), in addition to the C implied by the first stage.

Firm Problem

- Assume that firms $j \in [0, 1]$ produces using technology

$$Y(j) = AN(j)^{1-\alpha}$$

where $Y(j)$ is their output, A is their productivity, (assumed common to all firms) and $N(j)$ is the labour they hire with $0 < \alpha < 1$.

Firm Problem

- Objective of the firm is to maximise its profits

$$\max_{\{P(j)\}} P(j)Y(j) - TC(Y(j))$$

subject to $Y(j) = C(j)$ as given from the household's demand curve.

- $TC(Y(j))$ is the total cost of the firm given its level of production.

- The expression for total cost is given by

$$\begin{aligned} TC(Y(j)) &= WN(Y(j)) \\ &= W\left(\frac{Y(j)}{A}\right)^{\frac{1}{1-\alpha}} \end{aligned}$$

Roadmap

Household Solution: Stage 1

- Lagrangian for the first stage given by

$$\mathcal{L}_1 = \left(\int_0^1 C(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} + \lambda_1 \left[WN - \int_0^1 P(j)C(j)dj \right]$$

where notice that the income is taken as given.

- FOC

$$\frac{\partial \mathcal{L}_1}{\partial C(j)} = 0 \tag{1}$$

$$\Rightarrow \left(\frac{\epsilon - 1}{\epsilon} \right) \left(\frac{\epsilon}{\epsilon - 1} C(j)^{-\frac{1}{\epsilon}} \right) \left(\int_0^1 C(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{1}{\epsilon-1}} - \lambda_1 P(j) = 0$$

$$\Rightarrow C(j)^{\frac{1}{\epsilon}} C^{\frac{1}{\epsilon}} = \lambda_1 P(j)$$

which holds for each $j \in [0, 1]$.

Household Solution: Stage 1

- Notice then that if you study any two varieties k, j that

$$\left(\frac{C(j)}{C(k)}\right)^{-\frac{1}{\epsilon}} = \frac{P(j)}{P(k)} \tag{2}$$

  where the left-side is the marginal rate of substitution between two varieties and the right side is the price ratio.

Household Solution: Stage 1

- Recall we mentioned earlier that we want some aggregator across the variety prices: denote it by P.
- From equation (2) we get that

$$\begin{aligned}
\frac{C(j)}{C(k)} &= \left(\frac{P(j)}{P(k)}\right)^{-\epsilon} \\
\Rightarrow P(j)\frac{C(j)}{C(k)} &= P(j)\left(\frac{P(j)}{P(k)}\right)^{-\epsilon} \\
\Rightarrow P(j)C(j) &= C(k)P(k)^{\epsilon}P(j)^{1-\epsilon} \\
\Rightarrow \int_0^1 P(j)C(j)dj &= C(k)P(k)^{\epsilon}\int_0^1 P(j)^{1-\epsilon}dj \\
\Rightarrow C(k)P(k)^{\epsilon} &= \frac{\int_0^1 P(j)C(j)dj}{\int_0^1 P(j)^{1-\epsilon}dj}
\end{aligned}$$

Household Solution: Stage 1

- Then recall that $\int_0^1 P(j)C(j)dj = WN$. It follows then that

$$\Rightarrow C(k) = \frac{P(k)^{-\epsilon}WN}{\int_0^1 P(j)^{1-\epsilon}dj}$$
$$= \frac{P(k)^{-\epsilon}}{P^{-\epsilon}}\frac{WN}{P}$$
$$= \left(\frac{P(k)}{P}\right)^{-\epsilon} C$$

  where the second line follows from the definition

$$P = \left(\int_0^1 P(j)^{1-\epsilon}dj\right)^{\frac{1}{1-\epsilon}}$$

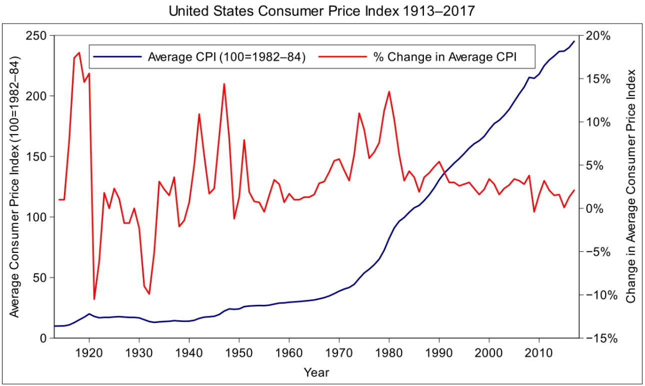
  and the third line comes from $PC = WN$.

- The demand curve for variety $\forall k \in [0,1]$ is $C(k) = \left(\frac{P(k)}{P}\right)^{-\epsilon} C$.

Household Solution: Stage 1

- What does the price index represent?

$$P = \left(\int_0^1 P(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$$



Household Solution: Stage 2

- Given the solution to the first stage, we solve the second

$$\mathcal{L}_2 = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} + \lambda_2 \left[WN - PC\right]$$

- FOC

$$\frac{\partial \mathcal{L}_2}{\partial C} = 0 \Rightarrow C^{-\sigma} - P\lambda_2 = 0$$
$$\frac{\partial \mathcal{L}_2}{\partial N} = 0 \Rightarrow -N^{\varphi} + W\lambda_2 = 0$$

or $N^{\varphi} C^{\sigma} = \frac{W}{P}$.

Firm Solution

- Firm optimally chooses its price subject to the demand curve. Objective

$$\widehat{\mathcal{L}} = P(j)\left(\frac{P(j)}{P}\right)^{-\epsilon} C - W\left(\frac{1}{A}\left\{\left(\frac{P(j)}{P}\right)^{-\epsilon} C\right\}\right)^{\frac{1}{1-\alpha}}$$

- FOC

$$\frac{\partial\widehat{\mathcal{L}}}{\partial P(j)} = 0$$

$$\Rightarrow (1-\epsilon)\left(\frac{P(j)}{P}\right)^{-\epsilon} C - \frac{(-\epsilon)}{1-\alpha}\frac{W}{A}\left(\frac{P(j)}{P}\right)^{-\epsilon-1}\frac{1}{P}C\times$$

$$\left(\frac{1}{A}\left\{\left(\frac{P(j)}{P}\right)^{-\epsilon} C\right\}\right)^{\frac{\alpha}{1-\alpha}} = 0$$

Firm Solution

- Simplify the FOC further

$$P(j)^{\frac{-(1-\alpha)-\alpha\epsilon}{1-\alpha}} = \frac{\epsilon-1}{\epsilon}(1-\alpha)\left(\frac{1}{W}\right)^{-1} P^{\frac{-\epsilon\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} C^{-\frac{1}{1-\alpha}}$$

$$\Rightarrow P(j) = \left\{\frac{\epsilon-1}{\epsilon}(1-\alpha)\left(\frac{1}{W}\right)^{-1} P^{\frac{-\epsilon\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} C^{-\frac{1}{1-\alpha}}\right\}^{\frac{1-\alpha}{-(1-\alpha)-\alpha\epsilon}}$$

which notice is identical across all varieties, (comes about given the assumption of A the same across all firms).

Firm Solution

- Exercise: show that $P(j)$ is a constant markup over marginal cost. That is

$$\begin{aligned} P(j) &= \frac{\epsilon}{\epsilon - 1} TC'(Y(j)) \\ &= \frac{\epsilon}{\epsilon - 1} \frac{1}{1-\alpha} \frac{W}{A} \left(\frac{Y(j)}{A} \right)^{\frac{\alpha}{1-\alpha}} \end{aligned} \tag{3}$$

where $TC'(Y(j))$ is marginal cost (derivative of the total cost).

Equilibrium Definition

- The general equilibrium of this simple static model is such that
  - All agents optimise,
  - The goods market clears for each variety $C(j) = Y(j) \ \forall j \in [0, 1]$.
  - The labour market clears $N = \int_0^1 N(j)dj$.

Aggregation

- Define aggregate output to be

$$Y = \left(\int_0^1 Y(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

which means that $Y = C$.

- Recall also that $P(j) = \widehat{P}$ was the same $\forall j \in [0,1]$. This means that

$$P = \left(\int_0^1 \widehat{P}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$$

$$\Rightarrow P = \widehat{P}$$

$$\Rightarrow Y(j) = Y$$

Roadmap

Wedge

- The presence of the monopolistic competition feature creates a distortion relative to first best.

- From equation (3) in addition to the aggregation conditions, see that

$$\frac{W}{P} = \frac{\epsilon}{\epsilon - 1} \underbrace{(1-\alpha)AN^{-\alpha}}_{\text{Marginal product of labour } (MP_N)}$$

- On the household-side, recall that

$$N^{\varphi} C^{\sigma} = \frac{W}{P}.$$

Wedge

- In general equilibrium, it follows that

$$N^{\varphi} C^{\sigma} = \frac{\epsilon}{\epsilon - 1} MP_N$$

- But notice that under the first best, (exercise: show),

$$N^{\varphi} C^{\sigma} = MP_N$$

which says that the MRS equals the marginal product of labour.

- The presence of markups creates a wedge between the MRS and MP_N.

Correcting the Distortion

- We can correct this distortion using fiscal subsidies.
- More on this later.

Roadmap

Takeaways

- This lecture, we studied the first ingredient into the new Keynesian model: monopolistic competition.
- Under this static framework, the optimal price of a firm equals a constant markup over its marginal cost, (no longer cost pricing).
- Markups create a distortion: can potentially be corrected with an employment subsidy.