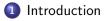
# Lecture 7: Theory of Corporate Finance V Agency Frictions: Moral Hazard

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3 Model Equilibrium



### Motivation

- How does a borrower's ability to steal or shirk affect the firm's ability to raise financing?
- If the lenders know that the borrower has these incentives, they may think twice before lending the firm money.









### Setup

- Consider a project, whose probability of success can be influenced by the borrower's effort level.
- For simplicity, assume that the model is static: only one time period.
- Investment takes place at the start of the period then returns realised at the end.
- If the borrower behaves, the probability of success is  $p_H$ .
- If the borrower misbehaves, the probability is  $p_L$ .
- The borrower has initial assets he can use for investment given by a > 0.
- In the case of success, an investment of size k yields gross return rk for r > 1 (i.e. proportional to scale of investment).
- In the case of failure, the project pays-out zero.



- The borrower gets a private benefit from misbehaving.
- Denote this private benefit by *bk* for *b* > 0 (again proportional to the scale of investment).

### Setup

- To finance an investment of size k, the borrower must borrow k a from creditors, (desired investment size less initial assets).
- They design the debt contract to be such that
  - Creditor receives payout r<sub>c</sub> in the case of success (creditor),
  - Creditor receives payout of zero in the case of failure.
- This means that the borrower receives
  - Borrower receives payout  $r_d$  in the case of success (debtor),
  - Borrower receives payout of zero in the case of failure.
- The payouts are defined such that  $r_d + r_c = rk$ .

### Setup

- We need to place some restrictions on the expected NPV per dollar of investment.
- Assume that positive expected NPV per unit in the case of behaviour

 $p_H r > 1$ 

which says the expected return for a unit of investment is bigger than the investment cost.

• But negative NPV per unit in the case of misbehaviour

$$1 > p_L r + b$$

which says that the overall expected return including the borrower's private benefit is less than the investment cost.

• For the project to be financed, we must incentivise the borrower to behave.









- We want to design the contract (choice of split between  $r_c$  and  $r_b$ ) to give the borrower incentive to behave.
- The incentive compatibility (IC) constraint is

$$p_{H}r_{b} \ge p_{L}r_{b} + bk$$

$$\Rightarrow (p_{H} - p_{L})r_{b} \ge bk$$

$$\Rightarrow r_{b} \ge \frac{bk}{p_{H} - p_{L}}$$
(1)

which says the borrower needs to get a payout in the case of success at least as large as the ratio of the private benefit from misbehaving relative to the probability change due to misbehaviour.

- We'll assume that the creditors break even in expectation as we did before.
- That is, the breakeven constraint is

$$p_H(rk - r_b) = k - a \tag{2}$$

which says the expected return the creditor gets is equal to the amount of financing they provide.

• See that re-arranging equation (2) gives

$$r_b = rk - rac{k-a}{p_H}$$

• Re-arranging (1) gives

$$k \le \frac{r_b(p_H - p_L)}{b} \tag{4}$$

(3)

• We can then combine equation (3) with inequality (4) to get

$$k \leq \frac{p_{H} - p_{L}}{b} \left[ rk - \frac{k - a}{p_{H}} \right]$$
$$\leq \frac{p_{H} - p_{L}}{b} rk - \frac{(p_{H} - p_{L})(k - a)}{bp_{H}}$$
$$\Rightarrow k \left[ 1 - \frac{p_{H} - p_{L}}{b} r + \frac{p_{H} - p_{L}}{bp_{H}} \right] \leq \frac{p_{H} - p_{L}}{bp_{H}} a$$
$$\Rightarrow k \leq \frac{(p_{H} - p_{L})a}{bp_{H} \left[ 1 - \frac{p_{H} - p_{L}}{b} r + \frac{p_{H} - p_{L}}{bp_{H}} \right]}$$
$$\leq \frac{a}{1 - p_{H} \left[ r - \frac{b}{p_{H} - p_{L}} \right]}$$
(5)

# Credit rationing

- Assume that  $1 p_H \left[ r \frac{b}{p_H p_L} \right] > 0$  for an interior solution. How do we interpret this? Exercise.
- What does inequality (5) say? Says that investment is constrained.
- Borrowing capacity is increasing in
  - Collateral of the borrower, a.
  - Return of successful project, r.
- Borrowing capacity is decreasing in private benefit of misbehaving, b.
- Probabilities of success? Exercise.

# Credit rationing

- Does this make sense?
- As *b* gets larger, the size of the agency conflict is increasing.
- Limits the extent of the overall investment that can take place through this borrowing limit.

# Credit rationing

- What does this mean for the welfare of the borrower?
- Lender breaks even, so all the NPV of the project accrues to the borrower.
- Borrower gets benefit of  $(p_H r 1)k$ . Why?
- He wants k to be as large as possible.
- The presence of this agency friction actually harms the borrower.
- The lender can get screwed if the borrower misbehaves.
- Lender passes-on this potential cost to the borrower through debt contract.

### Maximal borrowings

• The maximum borrowings that can be taken out are

$$k - a \leq \frac{p_{H}\left[r - \frac{b}{p_{H} - p_{L}}\right]}{1 - p_{H}\left[r - \frac{b}{p_{H} - p_{L}}\right]}a$$

• Again, increasing in collateral and decreasing in agency benefit.





3 Model Equilibrium



### Summary

- The borrower is incentivised to behave via the contract design scheme.
- In equilibrium he won't misbehave.
- Lender passes these agency costs on to the borrower through design of the debt contract.