Lecture 7: Theory of Corporate Finance V
Agency Frictions: Moral Hazard

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Roadmap

Motivation

- How does a borrower's ability to steal or shirk affect the firm's ability to raise financing?
- If the lenders know that the borrower has these incentives, they may think twice before lending the firm money.

Roadmap

Setup

- Consider a project, whose probability of success can be influenced by the borrower's effort level.
- For simplicity, assume that the model is static: only one time period.
- Investment takes place at the start of the period then returns realised at the end.
- If the borrower behaves, the probability of success is p_H.
- If the borrower misbehaves, the probability is p_L.
- The borrower has initial assets he can use for investment given by $a > 0$.
- In the case of success, an investment of size k yields gross return rk for $r > 1$ (i.e. proportional to scale of investment).
- In the case of failure, the project pays-out zero.

Setup

- The borrower gets a private benefit from misbehaving.
- Denote this private benefit by bk for $b > 0$ (again proportional to the scale of investment).

Setup

- To finance an investment of size k, the borrower must borrow $k - a$ from creditors, (desired investment size less initial assets).
- They design the debt contract to be such that
  - Creditor receives payout r_c in the case of success (creditor),
  - Creditor receives payout of zero in the case of failure.
- This means that the borrower receives
  - Borrower receives payout r_d in the case of success (debtor),
  - Borrower receives payout of zero in the case of failure.
- The payouts are defined such that $r_d + r_c = rk$.

Setup

- We need to place some restrictions on the expected NPV per dollar of investment.
- Assume that positive expected NPV per unit in the case of behaviour

$$p_H r > 1$$

which says the expected return for a unit of investment is bigger than the investment cost.

- But negative NPV per unit in the case of misbehaviour

$$1 > p_L r + b$$

which says that the overall expected return including the borrower's private benefit is less than the investment cost.

- For the project to be financed, we must incentivise the borrower to behave.

Roadmap

Contract design

- We want to design the contract (choice of split between r_c and r_b) to give the borrower incentive to behave.

- The incentive compatibility (IC) constraint is

$$\begin{aligned} p_H r_b &\geq p_L r_b + bk \\ \Rightarrow (p_H - p_L) r_b &\geq bk \\ \Rightarrow r_b &\geq \frac{bk}{p_H - p_L} \end{aligned} \tag{1}$$

  which says the borrower needs to get a payout in the case of success at least as large as the ratio of the private benefit from misbehaving relative to the probability change due to misbehaviour.

Contract design

- We'll assume that the creditors break even in expectation as we did before.
- That is, the breakeven constraint is

$$p_H(rk - r_b) = k - a \tag{2}$$

which says the expected return the creditor gets is equal to the amount of financing they provide.

Contract design

- See that re-arranging equation (2) gives

$$r_b = rk - \frac{k - a}{p_H} \tag{3}$$

- Re-arranging (1) gives

$$k \leq \frac{r_b(p_H - p_L)}{b} \tag{4}$$

Contract design

- We can then combine equation (3) with inequality (4) to get

$$
\begin{aligned}
k &\leq \frac{p_H - p_L}{b}\left[rk - \frac{k-a}{p_H}\right] \\
&\leq \frac{p_H - p_L}{b} rk - \frac{(p_H - p_L)(k-a)}{bp_H} \\
\Rightarrow k\left[1 - \frac{p_H - p_L}{b} r + \frac{p_H - p_L}{bp_H}\right] &\leq \frac{p_H - p_L}{bp_H} a \\
\Rightarrow k &\leq \frac{(p_H - p_L)a}{bp_H\left[1 - \frac{p_H - p_L}{b} r + \frac{p_H - p_L}{bp_H}\right]} \\
&\leq \frac{a}{1 - p_H\left[r - \frac{b}{p_H - p_L}\right]} \qquad (5)
\end{aligned}
$$

Credit rationing

- Assume that $1 - p_H \left[r - \frac{b}{p_H - p_L} \right] > 0$ for an interior solution. How do we interpret this? Exercise.
- What does inequality (5) say? Says that investment is constrained.
- Borrowing capacity is increasing in
  - Collateral of the borrower, a.
  - Return of successful project, r.
- Borrowing capacity is decreasing in private benefit of misbehaving, b.
- Probabilities of success? Exercise.

Credit rationing

- Does this make sense?
- As b gets larger, the size of the agency conflict is increasing.
- Limits the extent of the overall investment that can take place through this borrowing limit.

Credit rationing

- What does this mean for the welfare of the borrower?
- Lender breaks even, so all the NPV of the project accrues to the borrower.
- Borrower gets benefit of $(p_H r - 1)k$. Why?
- He wants k to be as large as possible.
- The presence of this agency friction actually harms the borrower.
- The lender can get screwed if the borrower misbehaves.
- Lender passes-on this potential cost to the borrower through debt contract.

Maximal borrowings

- The maximum borrowings that can be taken out are

$$k - a \leq \frac{p_H \left[r - \frac{b}{p_H - p_L} \right]}{1 - p_H \left[r - \frac{b}{p_H - p_L} \right]} a$$

- Again, increasing in collateral and decreasing in agency benefit.

Roadmap

Summary

- The borrower is incentivised to behave via the contract design scheme.
- In equilibrium he won't misbehave.
- Lender passes these agency costs on to the borrower through design of the debt contract.