

L5: Taxes

Adam Hal Spencer

The University of Nottingham

Essentials of Financial Economics 2021
Financial Decision-Making (1st Quarter)

Roadmap

- 1 Introduction
- 2 Debt Tax Shields
- 3 Adjusted Present Value (APV) Method
- 4 Weighted Average Cost of Capital (WACC)
- 5 Conclusion

Motivation

- *The only things that are certain in life are death and taxes.*
- Nobody likes them.
- A surprising twist: taxes actually make borrowing look *more attractive*.

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Tax deductibility of interest (1)

- Interest payments are tax deductible in the United Kingdom.
 - There are some caps to the amount you can claim.
- Paid out of **before** tax earnings.
- Holding all else equal, a higher level of interest payments reduces the firm's taxable income.
- Debt financing has an **additional benefit** over equity financing along this channel.

Tax deductibility of interest (2)

- Consider two firms — Firm Unlevered and Firm Levered.
- Firm Unlevered is 100% equity while Firm Levered borrowed \$1,000 worth of debt at 8% interest.
- Assume that capital expenditures, depreciation and the change in net working capital are zero.

	Firm Unlevered	Firm Levered
EBIT	500	500
Interest	0	80
Pretax income	500	420
Tax (35%)	175	147
Net income to shareholders	325	273
Total income to D and E	$325 + 0 = 325$	$273 + 80 = 353$
Tax shield from debt	0	28

- Total cash flow from Firm Levered is 28 higher — known as the **debt tax shield (DTS)**.

Tax deductibility of interest (3)

- Firm Unlevered is 100% equity while Firm Levered borrowed \$ D worth of debt at $r_D\%$ interest.

	Firm Unlevered	Firm Levered
EBIT	C	C
Interest	0	$r_D D$
Pretax income	C	$C - r_D D$
Tax (35%)	$\tau^C C$	$\tau^C (C - r_D D)$
Net income to shareholders	$(1 - \tau^C) C$	$(1 - \tau^C) (C - r_D D)$
Total income to D and E	$(1 - \tau^C) C$	$(1 - \tau^C) C + \tau^C r_D D$
Tax shield from debt	0	$\tau^C r_D D$

- Value from having debt of D at interest rate of r_D is $\tau^C r_D D$.

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Valuation of tax shields

- If a firm takes-out some debt worth D at an interest rate of r_D for T periods

$$PV(DTS) = \sum_{t=1}^T \frac{\tau^C r_D D}{(1 + r^*)^t}. \quad (1)$$

- If the debt is assumed to be held in perpetuity, then

$$PV(DTS) = \frac{\tau^C r_D D}{r^*}. \quad (2)$$

- What discount rate should we use for r^* in (1) and (2)?

Discount rate for tax shields

- What discount rate should we use for r^* in the tax shields formula?
- The rate should reflect the riskiness of the tax shield payments.
 - (1) r_D : reflects the risk of the debt that generates the tax shields.
 - (2) r_A : reflects the risk of the corporate profits, which we need to generate tax shields.
 - (3) r_E : gives a conservative estimate of the risk.
- When we set $r^* = r_D$ then the formula for (2) simplifies

$$PV(DTS) = \tau^C D$$

Adjusted present value method (APV) (1)

- A method for evaluating firms and projects.
- Say that a project is to be financed using debt **and** equity.
- APV method looks at a **counterfactual** where the project is 100% equity-financed then adds-in the tax benefits of debt.
- Explicitly separates the project's cash flows that are associated with **real** operations and **financing**.

Adjusted present value method (APV) (2)

- Refer back to slide (6) with Firm Unlevered and Firm Levered.
- Let's assume that their cash flows are perpetual.
 - I.e. each firm has project cash flows of C each period while Firm Levered has debt of D maintained in perpetuity.
- Denote the value of Firm Unlevered and Firm Levered by V_U and V_L respectively.

$$V_U = PV[(1 - \tau^C)C]$$

$$V_L = PV[(1 - \tau^C)C + \tau^C r_D D],$$

which can be combined to get

$$V_L = V_U + PV(DTS), \quad (3)$$

which is known as the **APV formula**.

- We'll make some more adjustments to equation (3) in future lectures.

Some notes on APV

- When calculating the APV of a new project, always use the **incremental** debt.
- τ^C is the **marginal** tax rate; not the average.
 - Need to look at the tax that you'll be charged on **marginal** debt.
- When we have positive taxes, we can create value by purely changing firm's capital structure, (see example later).

Personal taxes (1)

- Investors in the firm need to pay additional taxes, (e.g. capital gains and dividends).
- The cash flow to individual investors **net of all taxes** is what's important.

Time Period	C-Corporation		Individual		
	Ordinary Income (τ_c)	Capital Gains ($\tau_{c,cg}$)	Ordinary Income (τ_p)	Dividends ($\tau_{p,div}$)	Capital Gains ($\tau_{p,cg}$)
Pre-1981	46.0%	28.0%	70.0%	70.0%	28.0%
1982-1986	46.0%	20.0%	50.0%	50.0%	20.0%
1987	40.0%	28.0%	39.0%	39.0%	28.0%
1988-1990	34.0%	34.0%	28.0%	28.0%	28.0%
1991-1992	34.0%	34.0%	31.0%	31.0%	28.0%
1993-1996	35.0%	35.0%	39.6%	39.6%	28.0%
1997-2000	35.0%	35.0%	39.6%	39.6%	20.0%
2001-2002	35.0%	35.0%	38.6%	38.6%	20.0%
2003-	35.0%	35.0%	35.0%	15.0%	15.0%

Source: Scholes et al. (2005), Taxes and Business Strategy (3rd ed), Table 1.1, p. 12

Personal taxes (2)

- Denote τ^i the personal tax rate paid on **interest/ordinary income**.
- Denote τ^e the personal tax rate on **dividends**.

	Flow to debtholders	Flow to equityholders
Amount to distribute	1	1
Corporate taxes	0	τ^C
Income after corporate tax	1	$1 - \tau^C$
Personal taxes	τ^i	$(1 - \tau^C)\tau^e$
Investor flow after tax	$(1 - \tau^i)$	$(1 - \tau^C)(1 - \tau^e)$

- See that there is no corporate tax liability for interest paid to debtholders.
- Tax benefit to debt if $(1 - \tau^i) > (1 - \tau^C)(1 - \tau^e)$.
 - Has been the case historically.

Personal taxes (3)

- When debt is held in **perpetuity** then

$$PV(DTS) = D\tau^*$$

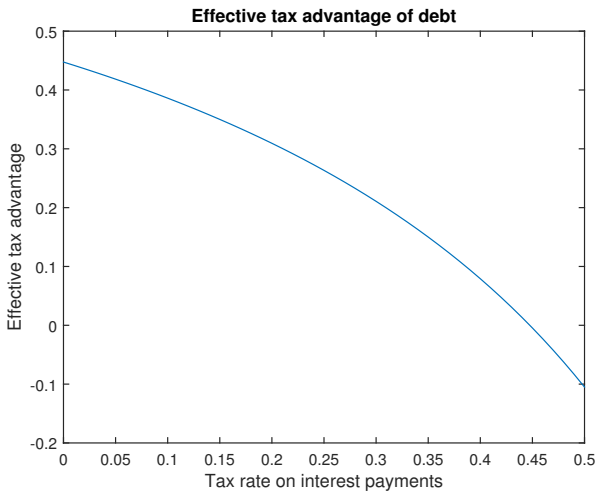
$$\tau^* = \frac{(1 - \tau^i) - (1 - \tau^C)(1 - \tau^e)}{1 - \tau^i},$$

which is expressed as a **percentage**. Derivation from Miller (1977).

- When τ^i is high, the tax advantage of debt is smaller.
- When $\tau^e = \tau^i$ then $PV(DTS) = \tau^C D$ as before.

Personal taxes (4)

- How does τ^* vary with τ^i ?
- Fix $\tau^C = 0.35$ and $\tau^e = 0.15$ and vary τ^i from 0 to 0.5 (below).



Example 1: leveraged recapitalisation

- Firm Pure has 1b shares outstanding, which are trading at \$640 per share.
- The firm is currently 100% equity.
- Assume that $\tau^C = 0.35$ and $r_D = 0.03$ while all personal tax rates are zero.
- Say that the firm issues \$100b in perpetual debt and uses the proceeds to **repurchase shares**.
 - (a) What happens to the share price? What is the value of the debt tax shield?
 - (b) What is the firm's value after the recap?
 - (c) How many shares are repurchased and at what price?
 - (d) Who gains and loses from the recap?
 - (e) What happens to the price at announcement?

Example 1 solution (1)

- The firm was originally 100% equity so $V_U = \$640 \times 1b = \$640b$.
- With the new debt, we can use APV to get

$$\begin{aligned} V_L &= V_U + PV(DTS) \\ &= \$640b + \$100(0.35)b \\ &= \$675b \end{aligned}$$

where I just assumed that the debt was perpetual and r_D was the appropriate discount rate.

- Use the two equations and two unknowns approach to find the new share price and the number of shares repurchased.

$$\begin{aligned} S \times P_{new} &= \$100b \\ P_{new} &= \frac{\$675b - \$100b}{1b - S} \end{aligned}$$

which can be solved to get $P_{new} = \$675$ and $S = \frac{100}{675}b$.

Example 1 [part 2]

- Now assume that there are personal tax rates of $\tau^e = 0.15$ and $\tau^i = 0.35$. Repeat the previous exercise!
- Give an intuition for the different price that the shares rise to on announcement relative to part 1.

Example 1 [part 2] solution (1)

- The effective tax benefit of debt is given by

$$\begin{aligned}\tau^* &= 1 - \frac{(1 - \tau^C)(1 - \tau^e)}{1 - \tau^i} \\ &= 0.15\end{aligned}$$

- Then the present value of the DTS is $PV(DTS) = \$15b$.
- $V_L = \$640b + \$15b = \$655b$.

$$\begin{aligned}SP_{new} &= \$100b \\ P_{new} &= \frac{655 - 100}{1 - S}\end{aligned}$$

which can be solved for $S = \frac{100}{655}$ and $P_{new} = \$655$.

- The price rise is now **smaller** given that there is less of a tax benefit of debt due to $\tau^i > 0$.

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Motivation

- We've explored the implications of taxes in the context of the APV method.
- APV is a versatile method of valuation, the further benefits of which we will explore in future lectures.
- But in industry, most firms will use a method called **weighted average cost of capital** (WACC) to account for tax benefits.
- APV method adjusted the **cash flows** associated with the project.
- WACC method instead adjusts the **discount rate**.

WACC definition (1)

- The APV method told us that we could **increase** firm value by assuming debt.
- When the corporate tax rate is positive, we define WACC as

$$\begin{aligned} \text{WACC} &= \frac{E}{D+E}r_E + \frac{D}{D+E}(1-\tau^C)r_D \\ &= r_A - r_D \frac{D}{V}\tau^C \end{aligned}$$

- How does this compare with r_A ?

$$\begin{aligned} r_A &= \frac{E}{D+E}r_E + \frac{D}{D+E}r_D \\ &> \frac{E}{D+E}r_E + \frac{D}{D+E}(1-\tau^C)r_D \\ &= \text{WACC} \end{aligned}$$

where the inequality relies on $\tau^C > 0$.

WACC definition (2)

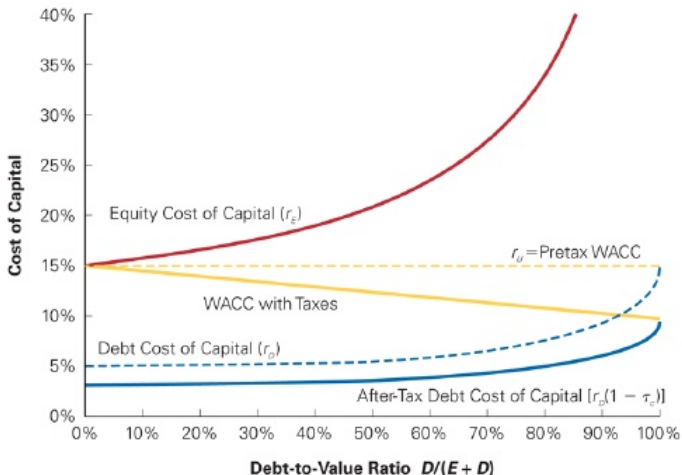
- How do we use the WACC estimate?
- If the cash flows from the **real** operations of the project are given by C_t for $t \in \{0, 1, 2, \dots\}$ then discount as follows

$$V_L = \sum_{t=0}^{\infty} \frac{C_t}{(1 + WACC)^t}$$

- Given that $WACC < r_A$, see that each of the cash flows C_t after discounting will be higher than using r_A .
- This incorporates the tax shields you'll be receiving from the Government!
- To use WACC though, we **must assume a constant leverage ratio!**
I.e. $\frac{D}{E}$ is constant!

WACC intuition

- A project must generate sufficient returns to compensate investors for risk.
- Interest payments reduce taxes and thus the required rate of return from the assets.



Equivalence of APV and WACC (1)

- Firm valuation under the two methods can be shown to be the same under certain conditions.
- Let's start by assuming that we have a firm that has the following characteristics:
 - Perpetual cash flow of C (after-tax) in each period.
 - Has a **constant** $\frac{D}{V_L}$ ratio.
 - Discounts its **tax shields** with r_A .

Example A [for Aston] (1)

- Aston Martin produces the Vanquish (Bond car).
- Assume the following
 - $r_D = 0.060$.
 - $r_E = 0.124$.
 - $\tau^C = 0.350$.
 - $D/A = 0.400$.



Example A [for Aston] (2)

- Assume that Aston Martin considers investing £12.5b in a new factory to be built in Cornwall, United Kingdom.
- Will generate perpetual cash flows of £1.731b before tax each period. (I.e. £1.125b after tax).
- Project has same risk as their current operations and will be financed with same debt and equity ratios.
 - (a) What is the project's WACC?
 - (b) What is the value of the project under the WACC method?

Suppose now instead that rather than financing the project using a fixed debt to equity ratio policy, that the firm will instead use fixed perpetual debt of £5b.

- (c) What is the value of the project under the APV method?

Example A [for Aston] solution (1)

(a) The project WACC is found as

$$\begin{aligned}WACC &= 0.124 \times 0.6 + 0.06 \times (1 - 0.35) \times 0.4 \\ &= 9\%.\end{aligned}$$

(b) The NPV using the WACC approach is then

$$\begin{aligned}NPV &= -12.5b + \frac{1.125b}{9\%} \\ &= 0.\end{aligned}$$

(c) Find the value of the unlevered firm as

$$\begin{aligned}r_A &= 12.4\% * 0.6 + 6\% * 0.4 \\ &= 9.84\% \\ \Rightarrow V_U &= -12.5 + \frac{1.125}{9.84\%} \\ &= -1.067b\end{aligned}$$

Example A [for Aston] solution (2)

- Then find the present value of the debt tax shields as

$$\begin{aligned}PV(DTS) &= \frac{5b \times 0.06 \times 0.35}{r} \\ &= \frac{0.105b}{r}\end{aligned}$$

which will vary depending on which r we choose.

- (i) Use $r_D = 0.06 \Rightarrow V_L = -1.067 + 1.75 = 0.685$.
- (ii) Use $r_A = 0.0984 \Rightarrow V_L = -1.067 + 1.067 = 0$.
- (iii) Use $r_E = 0.124 \Rightarrow V_L = -0.218$.

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Takeaways

- Taxes and capital structure: interest payments are a tax writeoff and so we generate **extra value** through tax shields.
- Two methods for evaluating — APV and WACC.
- WACC is the primary method of use in the real world.