Lecture 5: Cash in Advance

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Roadmap

Recap

- Last time we introduced money into the RBC model.
- Gave us some insights, but this approach is pretty cheap.
- CIA: assume that money is required to undertake certain types of transactions.
- Without money, these transactions cannot be made.
- Medium of exchange.
- Clower, R., (1967), "A Reconsideration of the Microfoundations of Monetary Theory", *Economic Inquiry*, 6, pp. 1–8.

When Cash is Mandatory for Transactions...

U.S. credit/debit cards don't work in Cuba...

Roadmap

Household Setup

- Households derive utility from consumption.
- Abstract from labour supply here.
- Three types of assets are held: cash, capital and riskless one period bonds.
- I.e. households own the capital stock here.
- Assume simple log utility over consumption each period.
- Deterministic model: abstract from any random shocks.
- Each period the household receives a lump-sum cash transfer from the government.

Goods

- Two types of goods: *cash* and *credit*.
- Cash goods are subject to the CIA constraint.
- Today we'll assume that consumption goods are for cash and capital are credit goods.

Timing

(1) Household enters time period t with state vector (m_t, b_t, k_t) of cash, bonds and capital respectively.

(2) Firms produce and goods market trades take place.

(3) Asset market opens and trades take place.

(4) Household leaves period t with state $(m_{t+1}, b_{t+1}, k_{t+1})$

Household Problem

- Household solves the problem

$$\max_{\{m_{t+1}, b_{t+1}, k_{t+1}, c_t\}} \sum_{t=0}^{\infty} \beta^t \log(c_t)$$

subject to

$$\begin{aligned} p_t c_t \leq & m_t + t_t \\ m_{t+1} + b_{t+1} + p_t(k_{t+1} - (1-\delta)k_t) \leq & m_t + t_t - p_t c_t \\ & + b_t(i_{t-1}) + \epsilon_t k_t + p_t d_t \end{aligned}$$

where the first constraint is the CIA constraint and the second is the asset market constraint.

- Why is there a price on investment?

Household Problem

- CIA constraint in real terms

$$c_t \leq \frac{\mu_t}{\pi_t} + \tau_t$$

where $\mu_t = \frac{m_t}{p_{t-1}}$, $\pi_t = \frac{p_t}{p_{t-1}}$ and $\tau_t = \frac{t_t}{p_t}$.

- Asset market constraint in real terms

$$\begin{aligned} \mu_{t+1} + \gamma_{t+1} + k_{t+1} - (1-\delta)k_t \leq & \frac{\mu_t}{\pi_t} + \tau_t - c_t \\ & + \frac{\gamma_t}{\pi_t}(i_{t-1}) + \iota_t k_t + d_t \end{aligned}$$

where $\gamma_{t+1} = \frac{b_{t+1}}{p_t}$ and $\iota_t = \frac{\epsilon_t}{p_t}$.

Firm Problem

- Firm solves the problem

$$\max_{\{k_{t+1}\}} p_t d_t = p_t k_t^{\alpha} - \epsilon_t k_t$$

Monetary Authority

- Money supply for the period equal to that from last plus the additional needed to cover the transfers

$$m_{t+1} = m_t + \tau_t$$

- Assume further that $\tau_t = gm_t$ for $g > 0$ for simplicity.

- Then

$$m_{t+1} = (1 + g)m_t$$

meaning that the money supply grows at a constant rate.

Roadmap

Household Optimality

- Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \log(c_t) + \sum_{t=0}^{\infty} \lambda_t^m \left[\frac{\mu_t}{\pi_t} + \tau_t - c_t \right] + \sum_{t=0}^{\infty} \lambda_t^a \times$$

$$\left[\frac{\mu_t}{\pi_t} + \tau_t - c_t + \frac{\gamma_t}{\pi_t}(i_{t-1}) + \iota_t k_t + d_t - \mu_{t+1} - \gamma_{t+1} - k_{t+1} + (1-\delta)k_t \right]$$

Household Optimality: FOCs

- FOCs

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Rightarrow \beta^t \frac{1}{c_t} - \lambda_t^m - \lambda_t^a = 0 \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{t+1}} = 0 \Rightarrow -\lambda_t^a + \lambda_{t+1}^m \frac{1}{\pi_{t+1}} + \lambda_{t+1}^a \frac{1}{\pi_{t+1}} = 0 \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = 0 \Rightarrow -\lambda_t^a + \lambda_{t+1}^a [\iota_{t+1} + (1 - \delta)] = 0 \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial \gamma_{t+1}} = 0 \Rightarrow -\lambda_t^a + \lambda_{t+1}^a \frac{i_t}{\pi_{t+1}} = 0 \tag{4}$$

Firm Optimality: FOCs

- FOCs

$$\frac{\partial p_t d_t}{\partial k_t} = 0 \Rightarrow \alpha p_t k_t^{\alpha-1} - \epsilon_t = 0 \qquad (5)$$
$$\Rightarrow \iota_t = \alpha k_t^{\alpha-1}$$

Canonical Representation

- Combine (1) and (2) money demand

$$\lambda_{a,t} = \beta^{t+1} \frac{1}{\pi_{t+1} c_{t+1}} \tag{6}$$

where $\lambda_{a,t} > 0$ means money is valued. Why?

- (3) and (4) give a no arbitrage condition

$$\iota_{t+1} + (1 - \delta) = \frac{i_t}{\pi_{t+1}} \tag{7}$$

which says the return on capital equals the real return on bonds. What happens if this doesn't hold?

Canonical Representation

- Euler equation

$$\beta^t \frac{1}{c_t} - \lambda_{m,t} = \left\{ \beta^{t+1} \frac{1}{c_{t+1}} - \lambda_{m,t+1} \right\} [\iota_{t+1} + (1 - \delta)] \qquad (8)$$

- If the CIA constraint is slack, this is our standard Euler equation.
- Say $\lambda_{m,t} > 0$ and $\lambda_{m,t+1} = 0$, what happens to (8)? What does this say about c_t relative to c_{t+1}?

Canonical Representation

- Bond demand from (4) and (2)

$$i_t = \frac{\lambda^m_{t+1} + \lambda^a_{t+1}}{\lambda^a_{t+1}} \tag{9}$$

- Resource constraint

$$c_t + k_{t+1} - (1-\delta)k_t = k_t^\alpha \tag{10}$$

- Money growth rule

$$\mu_{t+1} = (1+g)\frac{\mu_t}{\pi_t} \tag{11}$$

Equilibrium Definition

- The competitive equilibrium of the CIA model is a sequence $\{c_t, k_t, b_t, m_t, \tau_t\}_{t=0}^{\infty}$ and a sequence of prices $\{p_t, \epsilon_t, i_t\}_{t=0}^{\infty}$ such that the household optimises and markets clear.

Roadmap

Binding v.s. Non-Binding

- See from (9) that $i_t > 1 \iff \lambda^m_{t+1} > 0$.
- This means that the CIA constraint binds iff the net nominal interest rate is positive.
- If the opportunity cost of holding money is positive, we'll only hold enough to facilitate our purchases and no more.
- If $i_t = 1$ then the constraint is slack. Why?

Binding v.s. Non-Binding

- From the Fisher equation, see that

$$r_t = \frac{i_t}{\pi_{t+1}} \Rightarrow i_t = r_t \pi_{t+1} \tag{12}$$

- Using (9) and (12) gives that

$$\frac{\lambda^m_{t+1} + \lambda^a_{t+1}}{\lambda^a_{t+1}} = r_t \pi_{t+1} \tag{13}$$

which says that $\lambda_{m,t+1} > 0$ iff $r_t \pi_{t+1} > 1$. I.e. $r_t > \frac{1}{\pi_{t+1}}$.

- (13) says that CIA binds when the rate of return on money is dominated by that on real bonds.

Roadmap

Short v.s. Long Run in this Model

- There are no deviations once we reach steady state. Why?

What Does the Steady State Look Like?

- We'll get a steady state in real variables.
- What about nominal variables? E.g. m_t or p_t?
- Will the multipliers be constants? Why?

What Does the Steady State Look Like?

- See that (11) implies

$$\bar{\pi} = (1 + g)$$

the gross inflation rate equals the gross growth rate of money.

- We next look at two cases: (i) CIA constraint binds in ss and (ii) CIA constraint is slack in ss.

What Does the Steady State Look Like? Slack CIA

- Implies $\bar{i} = 1$.
- Equation (8) gives that

$$\frac{1}{\beta} = [\bar{\iota} + (1 - \delta)].$$

- Also see that (7) gives

$$\frac{1}{1+g} = [\bar{\iota} + (1 - \delta)].$$

- We can only have steady state here if $\beta = 1 + g$.
- But $\beta < 1$...what does this mean for $\bar{\pi}$ and why?

What Does the Steady State Look Like? Binding CIA

- Implies $\bar{i} > 1$.
- Equation (6) yields

$$\lambda_t^a = \beta^{t+1} \frac{1}{\bar{\pi}\bar{c}}$$

- (7) and (9) then say

$$\lambda_t^m = \beta^t \frac{1}{\bar{c}} \left\{ 1 - \frac{\beta}{\bar{\pi}} \right\} \tag{14}$$

What Does the Steady State Look Like? Binding CIA

- Equations (8) and (14) give that

$$
\begin{aligned}
&\beta^t \frac{1}{\bar{c}} - \beta^t \frac{1}{\bar{c}} \left\{1 - \frac{\beta}{\bar{\pi}}\right\} \\
&= \left\{\beta^{t+1} \frac{1}{\bar{c}} - \beta^{t+1} \frac{1}{\bar{c}} \left\{1 - \frac{\beta}{\bar{\pi}}\right\}\right\} [\bar{\iota} + (1 - \delta)] \\
\Rightarrow & 1 = \beta[\bar{\iota} + (1 - \delta)]
\end{aligned}
$$

which means that the capital stock is independent of nominal variables!

Affect of Money on Capital

- We don't need cash to buy capital.
- If we put capital goods into the CIA constraint, this independence result would break.
- **Punchline:** that's the problem with CIA, the results are greatly impacted by our assumptions on what goods require cash.

Roadmap

Summary

- CIA model seems a little more natural than MIU.
- Results can be similar depending on assumptions.
- Assume capital goods also require cash: soon on a problem set.