Lecture 3: Theory of Corporate Finance I
Modigliani & Miller Capital Structure Irrelevance Theorem

Adam Hal Spencer

The University of Nottingham

Advanced Financial Economics 2020

Roadmap

Motivation

- A company wants to invest. How should they pay for it?
- External financing: debt or equity?
- Internal financing: retained earnings?
- We'll think about this in the context of a basic two-period model.

Modigliani & Miller (1958) Theorem

- *The total value of the securities issued by a firm is independent of the firm's choice of capital structure. The firm's value is determined by its real assets and growth opportunities, not the type of securities it issues.*
- Means we can issue debt or equity or use internal funds; it really doesn't matter.
- Only holds in the absence of financial frictions.

Financial frictions

- The M&M (1958) theorem only holds when we have the following conditions simultaneously.

(1) Perfect and complete capital markets.

(2) No taxes.

(3) Bankruptcy is not costly.

(4) Capital structure doesn't affect investment decisions and cash flows.

(5) Symmetric information between insiders and outsiders.

- The negation of these assumptions are financial frictions.
- In the presence of financial frictions, firm value can in fact depend on capital structure.

Roadmap

Setup in $t = 0$

- Consider a world with two time periods $t \in \{0, 1\}$.
- A firm invests in $t = 0$ in productive capital (k).
- It needs to finance this investment by issuing external financing.
- Can issue new debt $(b > 0)$ or new equity $(e_0 < 0)$.
- Draws a stochastic (random) productivity shock

Setup in $t = 0$

- The lenders are assumed to demand an interest rate on the debt such that they break-even in expectation.
- I.e. the value of the funds they give the firm equal what they expect to receive back next period.

Setup in $t = 1$

- Draws a stochastic (random) productivity shock (θ) at the start of period $t = 1$.
- This shock is unknown to the firm at time $t = 0$.
- The shock can take one of two values $\theta \in \{0, 1\}$.
- Denote the probability of drawing $\theta = 1$ by $p \in [0, 1]$.
- If the firm has zero productivity the does not produce and thus defaults.
- After they choose to default, the capital stock is handed-over to the creditors, who liquidate it for ξk where $\xi \in [0, 1]$.
- If the firm defaults on its debt, the creditors (lenders) take control of the firm's assets.
- Assume that the capital stock fully depreciates after use.

Setup in $t = 1$

- The firm's pays a dividend to its owners in period $t = 1$ denoted by $e_1 \geq 0$.
- Weakly positive due to limited liability.
- The objective of the firm is to maximise the value to its equityholders, defined by $v = e_0 + \beta \mathbb{E}_\theta[e_1(\theta)]$ where $\beta \in [0, 1]$ is a discount factor.
- The expectation over $e_1(\theta)$ is with respect to the firm's productivity draw θ.
- Firm produces with production function $y = \theta k^\alpha$ where y is output, k is productive capital, θ is productivity and $\alpha \in [0, 1]$.

Roadmap

Firm's problem

- Firm maximises the expected value going to shareholders (owners).

$$\max_{k,b} \; v = e_0 + \beta \mathbb{E}_\theta [e_1(\theta)]$$
$$= e_0 + \beta [p e_1(\theta = 1) + (1-p) e_1(\theta = 0)]$$

where

$$e_0 = -k + b$$

$$e_1(\theta = 0) = 0$$

$$e_1(\theta = 1) = \max_{\{D,P\}} \Big[\underbrace{0}_{\text{Default (D)}} \;,\; \underbrace{k^\alpha - b(1+r)}_{\text{Produce and repay debt (P)}} \Big]$$

where D refers to discrete choice default and P is for choice produce.

Lender's problem

- The lender demands interest rate r such that

$$l_0 + \beta \mathbb{E}_\theta[l_1(\theta)] = 0$$

where

$$l_0 = -b$$

$$l_1(\theta = 0) = \xi k$$

$$l_1(\theta = 1) = \mathbb{1}_{D,\theta=1}[\xi k] + \mathbb{1}_{P,\theta=1}[b(1 + r)]$$

where $\mathbb{1}_{i,\theta=j}$ is an indicator function for the firm's choice at time $t = 1$ conditional on productivity $\theta = 1$.

- That is $\mathbb{1}_{D,\theta=1} = 1$ if the firm with productivity $\theta = 1$ and defaults.
- Why does the expression equal zero?

Simplifying the problems

- These indicator functions and max operators are nasty. Can we get rid of them?
- Will a firm that draws $\theta = 1$ ever default? No!
  - If they do, then the lender for sure will only get back ξk at $t = 1$.

    $\Rightarrow b = \beta \xi k$ by the lender's problem.

    $\Rightarrow v = e_0 = -k + \xi k < 0$ if $k > 0$ since $\xi \in [0, 1]$.

    $\Rightarrow k = 0$ since it's better to just invest nothing.

    $\Rightarrow b = 0$, meaning that they never borrowed anything in the first place.
- So we know that default happens for sure if $\theta = 0$ and the firm operates for sure if $\theta = 1$.

Simplifying the lender's problem

- Then we can re-write the lender's problem as

$$l_0 + \beta \mathbb{E}_\theta [l_1(\theta)] = 0$$

where

$$l_0 = -b$$

$$l_1(\theta = 0) = \xi k$$

$$l_1(\theta = 1) = b(1 + r).$$

Simplifying the lender's problem

- We can thus solve for r in the following equation

$$
\begin{aligned}
& -b+\beta\{pb(1+r)+(1-p)\xi k\}=0 \\
& \Rightarrow r=\frac{1}{p}\left[\frac{1}{\beta}-(1-p)\xi\frac{k}{b}\right]-1
\end{aligned}
$$

- Does this make sense?

- Says that the interest rate is an increasing function of leverage $\frac{b}{k}$.

Simplifying the firm's problem

- The objective function then becomes

$$\max_{\{b,k\}} v = -k + b + \beta\{p[k^\alpha - b(1+r)] + (1-p)(0)\}$$
$$= -k + b + \beta p[k^\alpha - b(1+r)]$$

subject to

$$r = \frac{1}{p}\left[\frac{1}{\beta} - (1-p)\xi\frac{k}{b}\right] - 1$$

- Why subject to the interest rate equation?
- Their choice of leverage affects the borrowing cost they are offered.

Solving the firm's problem

- Then we can take the derivative for investment as

$$\begin{aligned}\frac{\partial v}{\partial k} &= -1 + \beta p \left[\alpha k^{\alpha-1} - b\frac{\partial r}{\partial k}\right] \\ &= -1 + \alpha\beta p k^{\alpha-1} + b\beta p\frac{1}{p}(1-p)\xi\frac{1}{b} \\ &= -1 + \alpha\beta p k^{\alpha-1} + \beta(1-p)\xi\end{aligned}$$

Solving the firm's problem

- The derivative for borrowing is

$$
\begin{aligned}
\frac{\partial v}{\partial b} &= 1 - \beta p \left[(1+r) + b \frac{\partial r}{\partial b} \right] \\
&= 1 - \beta p \left[(1+r) + b \frac{1-p}{p} \xi \frac{k}{b^2} \right] \\
&= 1 - \beta p \frac{1}{\beta p} \\
&= 0.
\end{aligned}
$$

- This means that borrowing is indeterminate.

- This is the crucial result of M&M (1958).

- Says that the firm is indifferent to any level of debt.

The investment problem without debt

- What happens if we remove the debt choice from the problem?
- That is: if $\theta = 1$, the firm produces and if $\theta = 0$, the firm liquidates the capital stock.

The investment problem without debt

- Firm's problem is now

$$\max_k \hat{v} = -k + \beta[pk^\alpha + (1-p)\xi k]$$

which has derivative

$$\frac{\partial \hat{v}}{\partial k} = -1 + \alpha\beta k^{\alpha-1} + \beta(1-p)\xi$$

which is the same as the investment derivative with debt!

- Debt choice is indeterminate and has no impact on the firm's investment choices.

Roadmap

Financial frictions

- This is of course just a benchmark model.
- If there were no financial frictions, then corporate finance would not exist as a field.
- How does the firm's problem and solution change when we introduce these frictions one at a time?