Lecture 21: Finance Part III Financial Intermediation

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Roadmap



- 2 Model Environment
- 3 Social Planner's Problem

Implementing the Optimal Contract with Intermediation



Motivation

- A financial intermediary is an institution that brings lenders and borrowers together to facilitate financial transactions.
- Banks are the most obvious examples.
- We'll typically think of banks as taking liquid deposits (from households) and then making out illiquid loans (to businesses or other households).
- This maturity mismatch can create problems...





Nottingham University

- Me to a friend back in Australia: *no banks in Nottingham would take me since I don't have proof of address. But I found a dodgy one on campus that would accept a lower standard of proof called Santander.*
- Friend to me: *nice*, *you'd better be careful though man, they might go under.*

• Diamond-Dybvig (1983), "Bank Runs, Deposit Insurance and Liquidity", *Journal of Political Economy*.



Intuition

- The paper utilises a simple game-theoretic model where banks balance a tensions between efficient risk sharing and the possibility of runs.
- Depositors may get a bad shock (e.g. lose their job), which would require them to withdraw their funds from the bank.
- There end up being two equilibria: one where there is efficient risk sharing and everything is happy.
- Another where depositors all panic and run to withdraw their deposits.

Roadmap





3 Social Planner's Problem





Setup: Depositors

- Three time periods $t \in \{0, 1, 2\}$.
- Unit mass of ex-ante identical depositors and a single bank.
- Each depositor has endowment of 1 to invest at t = 0.
- Idiosyncratic shocks drawn by the depositors at t = 1.
 - Fraction s are impatient and want to consume at t = 1.
 - Fraction 1 s are patient and can consume either at t = 1 or t = 2.
 - An individual's type is private information but the fraction *s* is known publicly.
- Assume CRRA preferences $c^{1-\sigma}/(1-\sigma)$ with $\sigma \ge 1$.

Setup: Asset

- The bank takes the depositor's funds and invests them into an illiquid asset.
- Denote the returns from liquidating this asset at time $t \in \{1, 2\}$ by r_t .
- If you liquidate that asset at t = 1, you get no return, (i.e. $r_1 = 1$).
- If you liquidate at t = 2 you get a positive return, (i.e. $r_2 = R > 1$).
- The issue will be that some depositors will want to withdraw their funds at t = 1, (impatient types), before the asset has generated a positive return.

Roadmap







Implementing the Optimal Contract with Intermediation



- If a social planner could offer an optimal insurance contract, what would it look like?
- I.e. how much should impatient and patient depositors get to consume, (c₁ and c₂ respectively).
- Same as usual: maximise welfare subject to a resource constraint.
- Also need an incentive compatibility constraint since this is now a contracting problem with information asymmetry.

Solve the problem

$$\max_{c_1,c_2} s\left\{\frac{c_1^{1-\sigma}}{1-\sigma}\right\} + (1-s)\left\{\beta\frac{c_2^{1-\sigma}}{1-\sigma}\right\}$$

subject to

$$egin{aligned} & sc_1 + (1-s)rac{c_t}{R} \leqslant 1 \ & rac{c_1^{1-\sigma}}{1-\sigma} \leqslant eta rac{c_2^{1-\sigma}}{1-\sigma} \end{aligned}$$

where the first constraint is the resource constraint and the second is the incentive compatibility constraint.

Constraints

• Recall the two constraints were given by

$$egin{aligned} & sc_1 + (1-s)rac{c_t}{R} \leqslant 1 \ & rac{c_1^{1-\sigma}}{1-\sigma} \leqslant eta rac{c_2^{1-\sigma}}{1-\sigma} \end{aligned}$$

- Resource constraint says we divide-up the unit endowment between the early withdrawals and the (discounted) late withdrawals.
- Incentive compatibility constraint says that the patient households are better off waiting until t = 2 to consume than to consume at t = 1.

• Lagrangian given by

$$\begin{split} \mathcal{L} &= s \left\{ \frac{c_1^{1-\sigma}}{1-\sigma} \right\} + (1-s) \left\{ \beta \frac{c_2^{1-\sigma}}{1-\sigma} \right\} + \lambda_1 \left[1 - sc_1 - (1-s)\frac{c_2}{R} \right] + \\ &+ \lambda_2 \left[\beta \frac{c_2^{1-\sigma}}{1-\sigma} - \frac{c_1^{1-\sigma}}{1-\sigma} \right] \end{split}$$

• FOCs given by

$$\frac{\partial \mathcal{L}}{\partial c_1} = 0 \Rightarrow sc_1^{-\sigma} - \lambda_1 s - \lambda_2 c_1^{-\sigma} = 0$$

$$\frac{\partial \mathcal{L}}{\partial c_2} = 0 \Rightarrow (1 - s)\beta c_2^{-\sigma} - \lambda_1 (1 - s)\frac{1}{R} + \lambda_2 \beta c_2^{-\sigma} = 0$$

• Guess and verify that the incentive compatibility constraint is slack.

• That is:
$$\lambda_2 = 0 \Rightarrow \frac{c_1^{1-\sigma}}{1-\sigma} < \beta \frac{c_2^{1-\sigma}}{1-\sigma}$$
.

• Follow-through with the implications of this to get restrictions or a contradiction. The c₁ FOC says

$$\Rightarrow sc_1^{-\sigma} - \lambda_1 s = 0$$
$$\Rightarrow \lambda_1 = c_1^{-\sigma}$$

which can be substituted-into the c_2 FOC to get

$$\Rightarrow \beta c_2^{-\sigma} - [c_1^{-\sigma}] \frac{1}{R} = 0$$
$$\Rightarrow c_2 = c_1 \left(\frac{\beta}{R}\right)^{-\frac{1}{\sigma}}$$

- Assume that $\beta = 1$: so now no discounting.
- Notion of patience and impatience relates purely to this idea of whether they have to consume now or not.

$$c_2 = c_1 R^{\frac{1}{\sigma}}$$

where $c_2 > c_1$ given that $\sigma > 1$ (risk averse) and R > 1.

• Back-out c_1 and c_2 optimal from the resource constraint to get

$$c_{1} = \frac{1}{s + (1 - s)R^{\frac{1 - \sigma}{\sigma}}} \ge 1$$
$$c_{2} = \frac{R^{\frac{1}{\sigma}}}{s + (1 - s)R^{\frac{1 - \sigma}{\sigma}}} \le R$$

- This is good: insuring against risk.
- The autarky allocation is unit consumption for impatient at t = 1 and R consumption at t = 2 for patient depositors.
- Insurance: consumption smoothing better facilitated!

Roadmap









Implementing the Optimal Contract with Intermediation



Implementation

- Bank takes deposits (liquid liability) and invests them in asset (illiquid) with payoff R at date t = 2.
- Use the following deposit contract:
 - Take deposit of 1 at t = 0,
 - Pay r_1 to depositors who withdraw early at t = 1,
 - Pay r_2 to depositors who withdraw late at t = 2.
- Check feasibility:
 - Need sr_1 funds at t = 1.
 - The remaining $1 sr_1$ is divided up amongst patient depositors $r_2 = \max\left(0, R\frac{1-sr_1}{1-s}\right)$.

Implementation

- Set the early return $r_1 = c_1$ from the social planner's problem.
- From the resource constraint, you'll get that

$$r_2 = R \frac{1-r_1}{1-s}$$

meaning that we can implement the optimal contract using deposits!

Implementation

- This is only one Nash equilibrium of the deposit game.
- Unfortunately there is also a bank run Nash equilibrium.
- All types might panic at t = 1 and withdraw early.

- Suppose some fraction η withdraw at t = 1.
- Return at t = 2 then depends on η .

•
$$r_2(\eta) = \max\left[0, R\frac{1-\eta r_1}{1-\eta}\right].$$

- Impatient types will always withdraw due to their preferences, so $\eta \ge s$.
- Patient types also find it optimal to withdraw when

$$egin{aligned} &r_2(\eta) < r_1 \ &\Rightarrow \eta \geqslant rac{1}{r_1}rac{R-r_1}{R-1} \end{aligned}$$

where $\eta < 1 \iff r_1 > 1$.

- Since $r_1 > 1$, it follows that there are two Nash equilibria!
 - (i) Regular times: $\eta = s$ and $r_2(s) = c_2$ in the optimal contract.
 - (ii) Bank run: $\eta = 1$ and $r_2(1) = 0$.

Suspension of Convertibility

- If the bank can commit to stop letting-out withdrawals at *t* = 1, then there is no issue.
- Hard to be credible though...
- Deposit insurance by the government can achieve this!

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- We can, in principle, implement the optimal risk sharing contract that the social planner chooses using intermediation.
- The maturity mismatch can create problems though.
- We get another Nash equilibrium where a bank run takes place and it goes bust.