Lecture 19: Finance Part I
Asset Pricing

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Roadmap

Motivation

- How can we determine rates of return on assets?

Motivation

- Typical starting point is the Lucas (1978) “tree” model.
- We’ll take some stock (the tree), which produces dividends (fruit) for a representative household each period.
- The stock price will be a reflection of the expected fruit that the tree will bear in the future.

Roadmap

Setup

- Endowment economy.
- Assume that there is one stock in the economy (one fruit-bearing tree).
- The tree yields d_t units of consumption goods in each period.
- Fruits can't be stored.
- Trees can't be produced.

Setup

- Households can hold riskless one period bonds, (that are in zero net supply).
- The shares in the stock are in unit net supply.
- Household will determined how much to consume, hold in shares and bonds.
- Assume that the dividend process for the stock follows a Markov process $\pi(d', d)$.
- Uncertainty in this model is at the aggregate level and households can not insure each other.

Household Problem

- Household solves the optimisation problem

$$\max_{\{c_t, a_{t+1}, b_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to

$$p_t a_{t+1} + b_{t+1} + c_t \leqslant r_t^f b_t + (p_t + d_t) a_t$$

where r_t^f here denotes the riskless rate on the bonds and p_t is the price of the risky asset.

- Think of a_{t+1} as the fraction of the shares in the tree that the household chooses to hold.

- What's the numeraire in this model?

Roadmap

Household Problem Solution

- Household has Lagrangian

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} + \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_t [r_t^f b_t + (p_t + d_t) a_t - p_t a_{t+1} - b_{t+1} - c_t]$$

with FOCs

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Rightarrow \beta^t c_t^{-\sigma} - \lambda_t = 0$$
$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} = 0 \Rightarrow -p_t \lambda_t + \mathbb{E}_t (p_{t+1} + d_{t+1}) \lambda_{t+1} = 0$$
$$\frac{\partial \mathcal{L}}{\partial b_{t+1}} = 0 \Rightarrow -\lambda_t + (r_{t+1}^f) \lambda_{t+1} = 0$$

why no expectation on the riskless rate?

Household Problem Solution

- Euler equation for stock

$$p_t = \mathbb{E}_t\left[\beta\left(\frac{c_t}{c_{t+1}}\right)^{-\sigma}(p_{t+1}+d_{t+1})\right]$$
$$\Rightarrow 1 = \mathbb{E}_t\left[\beta\left(\frac{c_t}{c_{t+1}}\right)^{-\sigma}\frac{(p_{t+1}+d_{t+1})}{p_t}\right]$$
$$\Rightarrow 1 = \mathbb{E}_t[\mathcal{M}_{t+1}r_{t+1}]$$

where $\mathcal{M}_{t+1} = \beta\left(\frac{c_t}{c_{t+1}}\right)^{-\sigma}$ is the stochastic discount factor and $r_{t+1} = \frac{(p_{t+1}+d_{t+1})}{p_t}$ is the return on the risky asset.

Household Problem Solution

- Euler equation for bond should look familiar

$$1 = \mathbb{E}_t[\mathcal{M}_{t+1} r^f_{t+1}]$$

Market Clearing

- Clearing in all markets says that

$$\begin{aligned} a_{t+1} &= 1 \\ b_{t+1} &= 0 \\ c_t &= d_t \end{aligned}$$

which are the conditions for the stock market, bond market and consumption goods market.

- Notice that the first two market clearing conditions are coming from the representative household assumption.

- The goods market is coming from the endowment economy assumption

Competitive Equilibrium

- The competitive equilibrium here is defined as a sequence of allocations $\{c_t, a_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ and prices $\{p_t, r_t\}_{t=0}^{\infty}$ such that households optimise and market clear.

Roadmap

Euler Equations

- Recall that we have the two Euler equations: one for stocks and one for the riskless bond.

- Combining these two gives

$$1 = \mathbb{E}_t[\mathcal{M}_{t+1} r_{t+1}] = \mathbb{E}_t[\mathcal{M}_{t+1} r^f_{t+1}]$$

Covariance

- Recall from your statistics classes that

$$Cov(x, y) = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y].$$

- Then we can write that

$$\begin{aligned} 1 &= \mathbb{E}_t[\mathcal{M}_{t+1}(r_{t+1})] \\ &= Cov_t(\mathcal{M}_{t+1}, r_{t+1}) + \mathbb{E}[\mathcal{M}_{t+1}]\mathbb{E}_t[r_{t+1}] \\ \Rightarrow \mathbb{E}_t[r_{t+1}] &= \frac{1 - Cov_t(\mathcal{M}_{t+1}, r_{t+1})}{\mathbb{E}_t[\mathcal{M}_{t+1}]} \end{aligned}$$

Riskless Asset

- Say that the riskless bond offers a constant discount r^f.
- Follows that $Cov_t(\mathcal{M}_{t+1}, r^f) = 0$.
- Then gives that $r^f = \frac{1}{\mathbb{E}_t[\mathcal{M}_{t+1}]}$

Stock

- The stock however had a stochastic return governed by the Markov process.
- The expected return on the stock will be determined with how it is correlated with the stochastic discount factor.
- See from

$$\mathbb{E}_t[r_{t+1}] = \frac{1 - Cov_t(\mathcal{M}_{t+1}, r_{t+1})}{\mathbb{E}_t[\mathcal{M}_{t+1}]}$$

that the expected return is higher than r^f when the covariance is negative and lower than r^f when the covariance is positive.

Stock

- What does this covariance object actually mean?
- Recall that in our problem above $\mathcal{M}_{t+1} = \beta \left(\frac{c_t}{c_{t+1}} \right)^{-\sigma}$.
- That is – the SDF will be high when consumption growth between periods is low.
- Asset returns will be high when their returns are positively correlated with consumption growth.

Intuition

- Say there are two states: (1) consumption is high and (2) when consumption is low.
- Say there are two assets: (A) pays dividends in high times and (B) pays dividends in low times.
- The value of a dividend is the marginal utility of consumption in the corresponding state.
- Assets A that pays out in the good time is not valuable since the marginal utility is low.
- Asset B provides insurance by paying out when times are bad (low c).

Risk Premia

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