Lecture 18: New Monetarist Model Part III
Third Generation Models

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Roadmap

Motivation

- Last time we studied a model with prices and indivisible money: Trejos and Wright.
- Now we allow money to also be divisible.
- Lagos and Wright (2005).

Lagos and Wright (2005)

- The innovation of this paper was that they were able to generate a model where agents can hold any amount of cash, (no longer constrained to 0 or 1).
- Doing this while maintaining analytical tractability comes at a cost though.
- The distribution of cash holdings becomes degenerate each period: people hold the same amount of cash.

Roadmap

Setup

- Here we're back to discrete time.
- The trick the authors use is to have two subperiods in each time period.
- They call the first subperiod is called the day market and the second subperiod is referred to as the night market.

Aside

Setup

- Continuum of agents over [0,1].
- Discounting between periods at rate of β.
- Day market: decentralised, people bumping into each other randomly and pairwise, (as they did in the previous two lectures).
- Night market: centralised, (referred to as Walrasian).
- Agents consume and supply labour in day market (x, h) and night market (X, H).

Setup

- In addition to the two sub-periods, also need a special utility function for tractability.

$$U(x, h, X, H) = u(x) - c(h) + U(X) - H$$

Day Market Setup

- Several different varieties of day goods x.
- Each agent only consumes a subset of the goods on offer.
- Each agent produces his own variety using one-for-one labour production function.
- Just like before, agents don't consume their own goods.
- Coincidence probabilities are
  - Double coincidence: δ
  - Single coincidence: σ
  - No coincidence: $1 - \delta - 2\sigma$.
- Goods are divisible and non-storable.

Night Market Setup

- Centralised market with only one good.
- The price of money is denoted by ϕ_t: meaning that $1/\phi_t$ is the price of goods in terms of money.
- Again, goods are divisible and non-storable.

Money

- Agents can hold any quantity of money $m \geq 0$.
- Notice that there are two cross-sections we need to keep track of:
  - $F_t(\tilde{m})$ is the measure of agents with $m \leq \tilde{m}$ going into the day market.
  - $G_t(\tilde{m})$ is the measure of agents with $m \leq \tilde{m}$ going into the night market.
- Start by assuming that the amount of money is fixed at M.
- Implies that

$$\int m dF_t(m) = \int m dG_t(m) = M \ \forall t$$

- What's the difference between a distribution and a measure?

Roadmap

Notation

- Make the following notation definitions:
  - m: dollars of the buyer,
  - $\tilde{m}$: dollars of the seller,
  - $q_t(m, \tilde{m})$: quantity produced by the seller (bought by buyer),
  - $d_t(m, \tilde{m})$: dollars paid by the buyer to the seller,
  - $B_t(m, \tilde{m})$: payoff in double coincidence meetings,
  - $V_t(m)$: value function for an agent with m dollars when entering day market,
  - $W_t(m)$: value function for agent with m dollars when entering night market.

Day Market Value Function

$$V_t(m) = \underbrace{\alpha\sigma \int [u(q_t(m,\tilde{m})) + W_t(m - d_t(m,\tilde{m}))] dF_t(\tilde{m})}_{\text{Buying}}$$

$$+ \underbrace{\alpha\sigma \int [-c(q_t(m,\tilde{m})) + W_t(m + d_t(m,\tilde{m}))] dF_t(\tilde{m})}_{\text{Selling}}$$

$$+ \underbrace{\alpha\delta \int B_t(m,\tilde{m}) dF_t(\tilde{m})}_{\text{Bartering}}$$

$$+ \underbrace{(1 - 2\alpha\sigma - \alpha\delta) W_t(m)}_{\text{Not trading}}$$

Night Market Value Function

$$W_t(m) = \max_{X,H,m'} U(X) - H + \beta V_{t+1}(m')$$

$$\text{s.t. } X = H + \phi_t(m - m')$$

You can also think of the budget constraint as saying

$$\frac{1}{\phi_t}X + m' = m + \frac{1}{\phi_t}H$$

i.e. the value of your consumption plus new money holdings equals the value of your production plus old money holdings.

Single Coincidence Trades

- When one buyer wants what the other sells but not the other way around.
- Nash bargaining solution solves the problem

$$[u(q) + W_t(m-d) - W_t(m)]^{\theta}[-c(q) + W_t(\tilde{m}+d) - W_t(\tilde{m})]^{1-\theta}$$

subject to $d \leq m$ and $q \geq 0$.

Double Coincidence Trades

- When each agent wants what the other sells.
- Bargain over the amount to trade with each other and any extra cash to be exchanged.
- That is: bargain over $q_i, q_j \geq 0$ and $d \leq m$, (quantity traded from i, j and money exchanged).
- Solve the problem

$$\max_{\{q_i, q_j, \Delta\}} \left[u(q_j) - c(q_i) + W_t(m-d) - W_t(m)\right]^{\theta}$$
$$\left[u(q_i) - c(q_j) + W_t(\tilde{m}+d) - W_t(\tilde{m})\right]^{1-\theta}$$

where $d \leq m$ and $0 \leq d + \tilde{m}$.

Characterising the Equilibrium

- The money distribution at the start of the decentralised market=, $F_t(m)$, is degenerate.
- Happens since the centralised market re-sets everything: everyone leaves with same amount of money.
- No need to show this, but one condition for the existence of equilibrium is that

$$-\phi_t + \beta\phi_{t+1} \leq 0$$

otherwise it will always be better to just hold more money!

- The benefit to holding more cash will always exceed the cost.
- Friedman rule allowed: $\phi_t/\phi_{t+1} = \beta$, (inflation rate equals discount factor).

Roadmap

Constant Money Growth

- Assume that the central bank follows $M_{t+1} = (1+g)M_t$.
- Consider stationary equilibria, in which aggregate real balances are constant over time.

Inflation Inefficiency

- Possible to show that a higher steady state inflation rate implies lower output in the decentralised market.
- Higher inflation raises the opportunity cost of holding money: lower demand for real balances.
- Inflation inefficiencies decreasing in β: would hold more of it if you didn't discount the future.
- Inefficiencies are decreasing in θ, (the bargaining power of the buyer), investment with cost $-\phi$, which he could have spent on consumption goods. Won't get full return on investment unless $\theta = 1$.

Roadmap

Takeaways

- Lagos and Wright (2005) is the standard in this area.
- Problem is the assumptions required to keep it tractable.
- If you use a computer, you can get richer dynamics.