Lecture 17: New Monetarist Model Part II
Second Generation Models

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Advanced Monetary Economics 2018

Roadmap

Motivation

- Recall that the first generation model assumed 1 unit of good for 1 unit of money.
- What does this assume about prices?
- It exogenously sets the price of goods equal to one unit of money!
- If we allow the fraction of goods to be divisible, then we can get endogenous prices.

Roadmap

Bargaining

- If we're going to have endogenous prices, we need to think about agents who meet bargaining over what they'll be.
- One of the most commonly-used forms of rational bargaining in economic modelling is Nash bargaining.
- Something you'll study in a lot of detail in game theory classes.
- We know that economic transactions generate surplus.
- Nash bargaining tells us how that surplus will be split between the agents.

Primatives of a Two-Person Bargaining Model

- A set of players.
- A set of feasible agreements.
- A disagreement outcome: what will each player get if they can't reach an agreement.
- A bargaining solution that satisfies Nash's 4 axioms.

Axiomatic Bargaining

- Nash (1950) proposed that a bargaining outcome should satisfy the following axioms:
  - **(a)** Pareto efficiency: can't make one player better without making the other worse (leave nothing on the table).
  - **(b)** Symmetry: identical negotiators should get the same amount of surplus.
  - **(c)** Invariant to affine transformations: scaling the payoffs and disagreement points shouldn't matter.
  - **(d)** Independence of irrelevant alternatives: if the solution is an element of a subset X then the solution must be chosen from X.

Nash Bargaining Solution

- Nash (1950) showed that the unique solution that satisfies the four axioms is the pair of payoffs, which satisfy the following optimisation program.

$$\begin{aligned} \max_{v_1, v_2} \ & (v_1 - d_1)(v_2 - d_2) \\ \text{s.t.} \ & (v_1, v_2) \ \text{feasible} \\ & (v_1, v_2) \geqslant (d_1, d_2) \end{aligned}$$

  where (v_1, v_2) is the pair of outcomes to the bargaining problem and (d_1, d_2) are the outside options for the players.

- The objective is referred to as the Nash product.

Nash Bargaining Solution

- What factors influence the solution to this problem?
- What should said factors do to the solution intuitively?
- Feasible set.
- Disagreement.

Generalised Nash Bargaining

- What if we also want to allow for differential bargaining powers?
- Generalised Nash bargaining with weights (α_1, α_2).
- Higher weight means more power.
- Generalised solution solves the following program

$$\begin{aligned} \max_{v_1, v_2} \ & (v_1 - d_1)^{\alpha_1}(v_2 - d_2)^{\alpha_2} \\ \text{s.t.} \ & (v_1, v_2) \text{ feasible} \\ & (v_1, v_2) \geqslant (d_1, d_2) \end{aligned}$$

- Solution will also be a function of the bargaining weights relative to each other.

Roadmap

Setup

- Back to the money search model.
- Recall last lecture that we had some probability $y = \mathbb{P}\mathrm{r}({}_jW_i|{}_iW_j)$.
- Let's now set $y = 0$ and let the goods be divisible.
- For simplicity: people never barter, just exchange goods for cash.
- Same setup as before but now when agents meet, they'll bargain over how much good will be exchanged for a unit of money.

Exchange Rate

- One unit of money is exchanged for q units of goods.
- Or the price of a good is $\frac{1}{q}$ units of money.

Utility Functions and Production Costs

- Assume now that there is a variable production cost associated with the amount q that you produce, (recall was constant in last lecture).
- Utility of consumption is $u(q)$ and cost of production is $c(q)$.
- Assume that $u'(q) > 0$, $u''(q) < 0$, $c'(q) > 0$ and $c''(q) > 0$ for all $q > 0$.
- Place the following assumptions on the functional forms
  - $u(0) = c(0) = 0$.
  - $u'(0) > c(0)$.
  - $u'(0) > 0$ and $u''(0) \leqslant 0$.
  - $c'(0) > 0$ and $c''(0) \geqslant 0$.
  - Exercise: interpret these conditions.

Roadmap

Value Functions

- Value functions now given by

$$rV_1 = (1 - M)[u(Q) + V_0 - V_1]$$
$$rV_0 = M[V_1 - V_0 - c(Q)]$$

where both agents take the price $q = Q$ as given.

- We can solve here for $V_0(Q)$ and $V_1(Q)$ — i.e. as functions of Q.

Nash Product

- Can write the Nash product as

$$\Omega \equiv [u(q) + V_0(Q) - T_1]^{\alpha_1}[V_1(Q) - c(q) - T_0]^{\alpha_0}$$

  where T_k for $k \in \{0, 1\}$ are the outside options. What's going on here?

- Little qs versus big Qs???

Q v.s. q

- In their value functions, the agents take Q as given.
- Think of this as saying that the agents are taking the prices, at which all the other agents in the model trade, as given.
- The agents can affect their own trading prices, not the going market rates.

Surpluses in the Nash Product

- Buyer: gets instantaneous utility from consumption now and future value given market prices.
- Seller: gets disutility from production cost but future value from holding money with its going value.
- Then you subtract-out their respective outside options.

Full Problem

- The problem to solve is given by

$$\max_q [u(q) + V_0(Q) - T_1]^{\alpha_1} [V_1(Q) - c(q) - T_0]^{\alpha_0}$$

subject to

$$u(q) + V_0 \geqslant V_1$$
$$V_1 - c(q) \geqslant V_0$$

Optimality Conditions

- In this model, its traditional to assume that $T_0 = T_1 = 0$ and $\alpha_0 = \alpha_1 = 0.5$.
- Can be shown that the constraints will never bind when $y = 0$ as assumed.
- Then simplifies the objective function down to

$$\Omega = [u(q) + V_0(Q)]^{1/2}[V_1(Q) - c(q)]^{1/2}$$

with FOC

$$[V_1(Q) - c(q)]u'(q) - [u(q) + V_0(Q)]c'(q) = 0 \tag{1}$$

which is found by differentiating Ω with respect to q and setting the derivative equal to zero.

Optimality Conditions

- The optimality condition (1) then defines a function $q = e(Q)$.
- If other agents are giving Q units of output for a unit of money then a particular pair bargaining will agree to $q = e(Q)$.
- Equilibrium is a fixed point such that the function $e(Q)$ is equal to itself.
- Says that the conjecture of Q is consistent with the solution we found.

Fixed Points

- Turns out to be two fixed points to the problem $q = 0$ and $q = q^e > 0$.

Roadmap

Welfare

- Define the welfare function again to be the average

$$W = MV_1 + (1 - M)V_0.$$

- See that this simplifies to

$$rW = M(1 - M)[u(q) - c(q)]$$

Optimal Price

- The optimal (welfare-maximising) price q^* can be found at

$$u'(q^*) = c'(q^*)$$

Equilibrium v.s. Efficient Price

- Recall from the constraints of the Nash bargaining problem that

$$\begin{aligned} u(q) + V_0(q) &\geqslant V_1(q) \\ V_1(q) - c(q) &\geqslant V_0(q) \\ \Rightarrow u(q) + V_0(q) &> V_1(q) - c(q) \\ \Rightarrow u(q^e) + V_0(q^e) &> V_1(q^e) - c(q^e) \\ \Rightarrow \frac{u(q^e) + V_0(q^e)}{V_1(q^e) - c(q^e)} &> 1 \end{aligned}$$

where q^e is the equilibrium price.

Equilibrium v.s. Efficient Price

- Remember the FOC that describes the equilibrium (q^e) is given by

$$[V_1(q^e) - c(q^e)]u'(q^e) = [u(q^e) + V_0(q^e)]c'(q^e)$$
$$\Rightarrow u'(q^e) = \frac{u(q^e) + V_0(q^e)}{V_1(q^e) - c(q^e)} c'(q^e)$$
$$\Rightarrow u'(q^e) > c'(q^e)$$

- Notice that we can now say something about where q^e sits relative to q^* based on the shapes of u and c.

- Since u is concave and c is convex, it must be that q^e lies to the left of q^*.

- (Once the gradient of u gets smaller than that of c, it stays smaller).

Equilibrium v.s. Efficient Price

- So we know that $q^e < q^*$.
- Means that the price of goods in terms of money is too high.
- Why?....

Equilibrium v.s. Efficient Price

- ....discounting.
- If he seller could produce and then consume right away, he'd go for the amount q^*.
- But he gets money in exchange for the goods.
- Need to wait to spend it in the future when he meets someone with goods he wants to consume.

Roadmap

Takeaways

- We've extended the basic money search framework to a scenario with prices.
- In the market equilibrium, the price ends up being too high for goods.
- All because of this matching friction: not guaranteed to meet a seller who has the goods we desire!