Lecture 16: New Monetarist Model Part I
First Generation Models

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Roadmap

1. Introduction
2. Model Environment
3. Equilibrium Trading
4. Welfare Analysis
5. Conclusion

What's in the Name?

- New Monetarism is founded on the idea that money matters.
- Just like the "old" monetarism in this regard.
- The new Keynesian model we just finished with talked about monetary policy.
- But there was no money in the model!!!!
- Here we stress the microfoundations of money.

Randy Wright

- UW-Madison is one of the hot-spots in the world for this area of research.
- Ray B. Zemon Chair in Liquid Assets in Department of Finance and Professor in Department of Economics.

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Medium of Exchange

- The microfoundations focus on the idea that money is a medium of exchange.
- Three frictions that will make money essential here
  - **(1)** Double coincidence of wants problem.
  - **(2)** Long-run commitment can not be enforced.
  - **(3)** Agents are anonymous.
- Search theory.

Three Generations of Models

- As in the literature, we proceed here in three steps:
  - **(1)** 1 unit of money and 1 unit of good: Kiyotaki and Wright (1993).
  - **(2)** 1 unit of money and endogenous units of goods: Trejos and Wright (1995).
  - **(3)** Endogenous units of money and goods: Lagos and Wright (2005).

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Setup

- Continuum of anonymous agents over $i \in [0, 1]$.
- Infinite-lived and have discount rate $r > 0$.
- Individuals produce different varieties of goods over interval $i \in [0, 1]$.
- I.e. good i is produced by agent i.
- Goods are non-storable: generating the need for money.
- One unit of good costs agent $c > 0$ to produce.

Double-Coincidence of Wants Problem

- People produce goods, but don't want to consume their own goods.
- Want to consume the goods of others: motivates trade.
- Agents are going to "bump into each other" randomly in a market place.
- When agent i meets j, denote the probability that agent i wants to consume j's good by ${}_iW_j$.

Double-Coincidence of Wants Problem

- When an agent consume a good they get a constant level of utility, $u > c$.
- If i doesn't consume anything, he gets zero utility.
- Denote the matching probabilities as
  - $\mathbb{P}\mathrm{r}({}_iW_i) = 0$.
  - $\mathbb{P}\mathrm{r}({}_jW_i) = x$.
  - $\mathbb{P}\mathrm{r}({}_jW_i | {}_iW_j) = y$.

Money

- Each agent who has money has one unit.
- Exogenous amount $M \in [0, 1]$ of money in economy.
- Fiat money: agents don't get utility from holding.
- M agents are initially endowed with money randomly.
- Assume that agents holding money can't produce.

Matching

- Agent i matches with another with arrival time μ.
- When i meets j they decide whether or not to trade, then part company and re-enter the market.
- If an exchange takes place between i and j, it will take one of two forms
  - **(1)** Barter: i and j each exchange 1 unit of good for the other's (double coincidence).
  - **(2)** Purchase: only one agent wants the good made by the other and that same agent has a unit of money. 1 unit of money is exchanged for the 1 unit of good.

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Trading Strategies

- An individual will never accept a good in trade if he doesn't want to consume it since they're non-storable.
- If there is a double coincidence, they'll barter.
- Will money always have value?
- Answer turns out to be no!

Endogenous Trading Probabilities

- If you're holding money, what is the probability that you'll spend it?
- You meet someone with arrival rate α.
- He can only produce if he's not holding money: happens with probability $1 - M$.
- You like what he makes with probability x.
- Denote μ_0 the probability that a person without money will want to trade goods for money.
- Denote μ_1 the probability that a person with money will want to trade money for goods.

Endogenous Trading Probabilities

- Let $\mu = \mu_0 \mu_1$ denote the probability you will both want to trade.
- Notice that μ_0 and μ_1 are endogenous objects still to be determined.
- Money circulates if $\mu > 0$.

Value Functions

- If an agent has money

$$rV_1 = \alpha x(1 - M)\mu(u + V_0 - V_1)$$

- If an agent is without money

$$rV_0 = \alpha xy(1 - M)\mu(u - c) + \alpha xM\mu(V_1 - V_0 - c)$$

Value Functions

- Recall that α was the arrival rate.
- We can re-normalise the time units by letting $x\alpha = 1$.
- Instead of thinking about this as "per meeting", think of it as "per meeting where the other guy has something you want".
- Then simplifies the previous two value functions

$$rV_1 = (1 - M)\mu(u + V_0 - V_1)$$
$$rV_0 = y(1 - M)\mu(u - c) + M\mu(V_1 - V_0 - c)$$

Net Gains from Trading

- Net gain from trading goods for money defined as Δ_0 (check)

$$\begin{aligned}\Delta_0 &= V_1 - V_0 - c \\ &= \frac{(1-M)(\mu - y)(u-c) - rc}{r+\mu}\end{aligned}$$

- Net gain from trading money for goods is defined as Δ_1 (check)

$$\begin{aligned}\Delta_1 &= u + V_0 - V_1 \\ &= \frac{(M\mu + (1-M)y)(u-c) + ru}{r+\mu}\end{aligned}$$

Equilibrium Trading Strategies

- We can then look at Δ_0 and Δ_1 to find the equilibrium trading probabilities for money holders and non-money holders.
- Agent of type $k \in \{0, 1\}$ will benefit from trading if $\Delta_k > 0$.
- Agent of type $k \in \{0, 1\}$ will not benefit from trading if $\Delta_k < 0$.
- Agent of type $k \in \{0, 1\}$ will be indifferent to trading if $\Delta_k = 0$.

Equilibrium Trading Strategies

- So we can find the endogenous trading probabilities as

$$\mu_k \begin{cases} = 1 \text{ if } \Delta_k > 0 \\ \in [0,1] \text{ if } \Delta_k = 0 \\ = 0 \text{ if } \Delta_k < 0 \end{cases}$$

what does the $\mu_k \in [0,1]$ mean? Why?

Finding the Equilibria

- Going forward we'll focus on finding pure strategy equilibria.
- That is: we'll see if $\mu_k = 1$ or $\mu_k = 0$ only.
- Also possible to find equilibria where $\mu_k \in [0, 1]$, but we'll not worry about this.

Equilibrium Trading Strategies: with Money

- See that $\mu_1 > 0$ regardless of μ_0 as

$$\Delta_1 = \frac{(M\mu + (1-M)y)(u-c) + ru}{r+\mu} > 0$$

since $u > c$ by assumption.

- The agent already has cash, he can only do better by getting rid of it since its fiat money.

Equilibrium Trading Strategies: without Money

- See that $\mu_0 > 0$ is ambiguous!
- Notice that

$$\Delta_0 = \frac{(1-M)\mu(u-c) - rc}{r+\mu} - \frac{(1-M)y(u-c) + rc}{r+\mu}.$$

- Given that $\mu_1 = 1$, follows that $\mu = \mu_0$.
- Can then see that Δ_0 has the same sign as

$$\mu_0 - \hat{\mu}_0$$

  where $\hat{\mu}_0 = \frac{rc+(1-M)y(u-c)}{(1-M)(u-c)}$.
- The agent without the money has the goods: they need to weigh-up whether taking the money is worth it or not.

Non-Monetary Equilibrium

- Will $\mu_0 = 0$ ever be an equilibrium?
- Check that $\Delta_0 < 0$ when $\mu_0 = 0$ to verify if it's equilibrium behaviour.
- See then that $\Delta_0 < 0$ with $\mu_0 = 0$ when

$$0 - \frac{rc + (1-M)y(u-c)}{(1-M)(u-c)} < 0$$
$$\Rightarrow c > \frac{(1-M)yu}{r + (1-M)y}.$$

- This is called a non-monetary equilibrium.

Monetary Equilibrium

- Will $\mu_0 = 1$ ever be an equilibrium?
- Check that $\Delta_0 > 0$ when $\mu_0 = 1$ to verify if it's equilibrium behaviour.
- See then that $\Delta_1 > 0$ with $\mu_0 = 1$ when

$$1 - \frac{rc + (1-M)y(u-c)}{(1-M)(u-c)} > 0$$

$$\Rightarrow c < \frac{(1-M)(1-y)}{r + (1-M)(1-y)}u$$

- This is called a monetary equilibrium.

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Welfare

- Let's look at average utility as our welfare metric.
- Recall: fraction M agents have money and fraction $1 - M$ don't have money.
- Define welfare as

$$\begin{aligned} W &= MV_1 + (1 - M)V_0 \\ \Rightarrow rW &= M(rV_1) + (1 - M)(rV_0) \\ &= M\{(1 - M)\mu(V_0 - V_1)\} \\ &\quad + (1 - M)\{y(1 - M)\mu(u - c) + M\mu(V_1 - V_0 - c)\} \\ &= (1 - M)[M\mu + (1 - M)y](u - c) \end{aligned}$$

Welfare

- See that rW is increasing in μ.
- It we're in a monetary equilibrium, how much money should be on issue?
- Set $\mu = 1$ and maximise the objective with respect to M.

$$rW = (1 - M)[(1 - M)y + M](u - c)$$

- Easy to show that

$$M^* = \frac{1 - 2y}{2 - 2y} \text{ if } y < 1/2$$
$$M^* = 0 \text{ if } y \geqslant 1/2$$

- If probability agents want each other's goods is sufficiently high then no need for money.

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Takeaways

- First generation: 1 unit of good and 1 unit of money.
- Microfounding money holdings based on the medium of exchange role.
- Allowed us to talk, in a very basic way, about optimal monetary policy!
- Next time: 1 unit money and endogenous goods.