Lecture 15: Empirical Literature in Asset Pricing

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Roadmap

Motivation

- Again, recall our consumption Euler equation from a few lectures ago

$$1 = \beta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} r_{t+1} \right] \tag{1}$$

- Last lecture, we built-off this relationship to say something about predicted returns in a regression context.

- Now we ask the opposite question: given returns data and this relationship, what can we say about investor preferences?

- Boils-down to mapping data into risk preference parameter σ through equation (1).

Roadmap

Data

- What do data say about risk preferences given our model?
- Mehra and Prescott (1985): asset return data pose a puzzle for the theory.
- Let's look at some data. In recent years in the U.S.
    - Standard deviation on risky stocks: 0.167,
    - Standard deviation of consumption growth: 0.0360,
    - Correlation of stock returns with consumption growth: 0.4,
    - Mean equity premium: 0.0618.
- These data are going to destroy our consumption-based asset pricing theory...

Model

- Let's try to derive an expression for σ in terms of the other parameters and variables given in (1).
- Start by re-writing in terms of net rates.
- Define

$$(1 + \Delta c_{t+1}) \equiv \frac{c_{t+1}}{c_t}$$
$$1 + \tilde{r}_{t+1} \equiv r_{t+1}$$

where we call Δc_{t+1} the net growth rate in consumption and $\tilde{r}_{t+1}$ is the net return.

Model

- Then re-write the Euler equation as

$$\frac{1}{\beta} = \mathbb{E}_t[(1 + \Delta c_{t+1})^{-\sigma}(1 + \tilde{r}_{t+1})]$$

Model

- Remember what a Taylor approximation is?
- If $f(x)$ is differentiable at point a, then we can approximate $f(x)$ through series

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots$$

where as the number of terms on the right-side gets higher, the approximation gets better.

Model

- If we have two variables, a 2^{nd} order Taylor expansion about the point (x_0, y_0) takes the form

$$f(x,y) \approx f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$
$$+\frac{1}{2}\left\{f_{xx}(x_0,y_0)(x-x_0)^2 + f_{xy}(x_0,y_0)(x-x_0)(y-y_0) + f_{yy}(x_0,y_0)(y-y_0)^2\right\}$$

Model

- Perform a second-order Taylor expansion on the Euler equation.
- Define

$$f(\Delta c_{t+1}, \tilde{r}_{t+1}) = (1 + \Delta c_{t+1})^{-\sigma}(1 + \tilde{r}_{t+1}).$$

Then see that derivatives are given by

$$\begin{aligned}
f_{\Delta c_{t+1}}(\Delta c_{t+1}, \tilde{r}_{t+1}) &= -\sigma(1 + \Delta c_{t+1})^{-\sigma-1}(1 + \tilde{r}_{t+1}) \\
f_{\tilde{r}_{t+1}}(\Delta c_{t+1}, \tilde{r}_{t+1}) &= (1 + \Delta c_{t+1})^{-\sigma} \\
f_{\Delta c_{t+1}\Delta c_{t+1}}(\Delta c_{t+1}, \tilde{r}_{t+1}) &= -\sigma(-\sigma - 1)(1 + \Delta c_{t+1})^{-\sigma-2}(1 + \tilde{r}_{t+1}) \\
f_{\tilde{r}_{t+1}\tilde{r}_{t+1}}(\Delta c_{t+1}, \tilde{r}_{t+1}) &= 0 \\
f_{\Delta c_{t+1}\tilde{r}_{t+1}}(\Delta c_{t+1}, \tilde{r}_{t+1}) &= -\sigma(1 + \Delta c_{t+1})^{-\sigma-1}
\end{aligned}$$

Model

- Expand around the point $(0, 0)$:

$$f(\Delta c_{t+1}, \tilde{r}_{t+1}) \approx -\sigma \Delta c_{t+1} + \tilde{r}_{t+1}$$
$$- \frac{1}{2} \left\{ \sigma(-\sigma - 1)[\Delta c_{t+1}]^2 + 2\sigma[\Delta c_{t+1}][\tilde{r}] \right\}$$

Then taking the expectation of this object yields

$$\mathbb{E}[f(\Delta c_{t+1}, \tilde{r}_{t+1})] \approx -\sigma \mathbb{E}[\Delta c_{t+1}] + \mathbb{E}[\tilde{r}_{t+1}]$$
$$+ \frac{1}{2}\sigma(\sigma + 1)\mathbb{E}\{[\Delta c_{t+1}]^2\} - \sigma \mathbb{E}\{[\Delta c_{t+1}][\tilde{r}]\}$$

Model

- Then remember the definitions of all these statistical objects

$$Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$
$$Cov(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y].$$

If $\mathbb{E}[X]$ and $\mathbb{E}[Y]$ are sufficiently close to zero, then we can say that

$$Var(X) \approx \mathbb{E}[X^2]$$
$$Cov(X, Y) \approx \mathbb{E}[XY]$$

Model

- Applying these definitions to our Taylor expansion then yields

$$\mathbb{E}[f(\Delta c_{t+1}, \tilde{r}_{t+1})] \approx -\sigma\mathbb{E}[\Delta c_{t+1}] + \mathbb{E}[\tilde{r}_{t+1}] + \frac{1}{2}\sigma(\sigma+1)Var[\Delta c_{t+1}] - \sigma Cov(\Delta c_{t+1}, \tilde{r}_{t+1}).$$

Then plugging this into the Euler equation gives

$$-\sigma\mathbb{E}[\Delta c_{t+1}] + \mathbb{E}[\tilde{r}_{t+1}] + \frac{1}{2}\sigma(\sigma+1)Var[\Delta c_{t+1}] - \sigma Cov(\Delta c_{t+1}, \tilde{r}_{t+1}) = \frac{1}{\beta}$$
$$\Rightarrow \mathbb{E}[\tilde{r}_{t+1}] = \frac{1}{\beta} + \sigma\mathbb{E}[\Delta c_{t+1}] + \sigma Cov(\Delta c_{t+1}, \tilde{r}) - \frac{1}{2}\sigma(\sigma+1)Var[\Delta c_{t+1}]$$

Model

- When we're thinking about the riskless rate (r^f), see that

$$r^f = \frac{1}{\beta} + \sigma \mathbb{E}[\Delta c_{t+1}] - \frac{1}{2}\sigma(\sigma + 1) Var[\Delta c_{t+1}]$$

given that r^f is non-stochastic. The premium for a risky asset can then be written as

$$\mathbb{E}[\tilde{r}_{t+1}] - r^f = \sigma Cov(\tilde{r}_{t+1}, \Delta c_{t+1}).$$

Model

- The Sharpe ratio is a measure of the risk-return tradeoff.
- Defined as the excess return of a risky asset over the riskless rate, normalised by the standard deviation of the risky returns.
- See this is given by

$$\frac{\mathbb{E}[\tilde{r}_{t+1}] - r^f}{Sd(\tilde{r}_{t+1})} = \sigma \frac{Cov(\tilde{r}_{t+1}, \Delta c_{t+1})}{Sd(\tilde{r}_{t+1})}$$
$$= \sigma Corr(\tilde{r}_{t+1}, \Delta c_{t+1}) Sd(\Delta c_{t+1})$$

given the definition of the correlation coefficient $Corr(x, y) = Cov(x, y)/[Sd(x)Sd(y)]$

Model

- Follows then that the coefficient of relative risk aversion implied by the model is

$$\sigma = \frac{\mathbb{E}[\tilde{r}_{t+1}] - r^f}{Sd(\tilde{r}_{t+1})} \frac{1}{Corr(\tilde{r}_{t+1}, \Delta c_{t+1}) Sd(\Delta c_{t+1})}$$

- Recall the data from earlier and see that they imply

$$\begin{aligned} \sigma &= \frac{0.0618}{0.167} \frac{1}{0.4 \times 0.036} \\ &= 25.7 \end{aligned}$$

- This is a counterfactually enormous number.

- Means that investors so risk averse that they're too afraid to leave their houses each day...

Model

- Means that our model is off!

Roadmap

Epstein-Zin Preferences

- Let's try alternative specifications of the investor preferences.
- Issue: CRRA utility like $\frac{c^{1-\sigma}}{1-\sigma}$ impose that the coefficient of relative risk aversion and inter-temporal elasticity of substitution are both governed exclusively by σ.
- Means that if an investor dislikes risk (variation in consumption across states for a fixed time), then they will also dislike variation in consumption across time.
- Not obvious that should be the case.

Epstein-Zin Preferences

- Alternative preferences that separate these two objects:

$$U_t = \left[c_t^{1-\rho} + \beta(\mathbb{E}_t[U_{t+1}^{1-\alpha}])^{(1-\rho)/(1-\alpha)}\right]^{\frac{1}{1-\rho}}$$

  where α is the coefficient of risk aversion and ρ^{-1} is the intertemporal elasticity of substitution.

- This is known as recursive utility. Why?

Epstein-Zin Preferences

- When $\alpha = \rho$, this simplifies-down to CRRA preferences (don't expect you to show this).
- What's the problem with these preferences?
- They're a mess!
- Also require assumptions on the evolution of the consumption process over time to get first order conditions in terms of observables.

Habit Formation

- Habits: gets at the idea that it's not your absolute level of consumption that gives utility, but rather the change on periods.
- Utility would be defined as

$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{(c_{t+s} - \lambda c_{t+s-1})^{1-\sigma}}{1-\sigma}$$

  where $\lambda > 0$ is a parameter that captures the effect of past consumption.
- Has the effect of making the household averse to consumption risk, even when σ is small.
- Small changes in consumption can give rise to large changes in the marginal utility of consumption.
- Implied σ is smaller.

Idiosyncratic and Uninsurable Income Risk

- Say that investors face some probability of losing their job.
- Assume that they are unable to insure against this possibility.
- Equities pay less generally in times when people are more likely to lose their jobs! Procyclical returns with business cycles.
- Equity premium is then the extra return needed to make holding equities palatable for investors.

Roadmap

Conclusion

- This asset pricing model is just that — a model.
- We can infer something about the reliability of the model by comparing its predictions with data.