Lecture 14: Empirical Methods II
Factor Models

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Advanced Financial Economics 2020

Roadmap

1. Introduction
2. Single-Factor Model
3. Multi-Factor Models
4. Conclusion

Motivations

- Recall that consumption CAPM that we derived last class

$$\mathbb{E}_t[r_{t+1}] - r^F_{t+1} = -\frac{1}{\mathbb{E}_t\left[(c_{t+1})^{-\sigma}\right]} Cov\left((c_{t+1})^{-\sigma}, r_{t+1}\right) \quad (1)$$

  which says that the excess return on a risky asset depends on its co-movements with consumption.

- Can we take this to data to say something about expected returns empirically?

- This equation forms the basis for an empirical tool known as factor models.

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Derivation

- We seek to manipulate the Euler equation for the asset such that we get a regression specification that we can take to the data.
- The covariance term in equation (1) looks an awful lot like a regression coefficient!
- Let's simplify our lives and assume that $\sigma = 1$: meaning that the utility function is logarithmic ($\log(c_t)$).

Derivation

- Start with the Euler equation for an arbitrary asset (recall $\sigma = 1$)

$$1 = \mathbb{E}_t \left[\beta \left(\frac{c_t}{c_{t+1}} \right) r_{t+1} \right] \tag{2}$$

- Notice that this also holds for the riskless asset

$$1 = \mathbb{E}_t \left[\beta \left(\frac{c_t}{c_{t+1}} \right) r^F_{t+1} \right] \tag{3}$$

- Subtract equation (3) from (2) to obtain

$$0 = \mathbb{E}_t \left[\left(\frac{c_t}{c_{t+1}} \right) \{ r_{t+1} - r^F_{t+1} \} \right] \tag{4}$$

Derivation

- Now notice that $\frac{c_t}{c_{t+1}} = 1 - \frac{c_{t+1} - c_t}{c_{t+1}}$.

- Substitute this into equation (4) to get

$$0 = \mathbb{E}_t \left[\left(1 - \frac{c_{t+1} - c_t}{c_{t+1}} \right) \left\{ r_{t+1} - r^F_{t+1} \right\} \right]$$

$$\mathbb{E}_t[r_{t+1}] - r^F_{t+1} = \mathbb{E}_t \left[\left(\frac{c_{t+1} - c_t}{c_{t+1}} \right) \left\{ r_{t+1} - r^F_{t+1} \right\} \right]. \qquad (5)$$

Derivation

- What is $\frac{c_{t+1}-c_t}{c_{t+1}}$?
- It's not exactly consumption growth....but pretty close.
- Approximate it with $\frac{c_{t+1}-c_t}{c_t}$. Then (5) becomes

$$\mathbb{E}_t[r_{t+1}] - r^F_{t+1} = \mathbb{E}_t\left[\left(\frac{c_{t+1}-c_t}{c_t}\right)\{r_{t+1} - r^F_{t+1}\}\right]. \qquad (6)$$

Derivation

- Imagine that there exists an asset in the market that delivers the exact same returns as consumption growth.
- Let's denote the return on this asset $r^C_{t+1} = \frac{c_{t+1} - c_t}{c_t}$.
- Then (6) becomes

$$\mathbb{E}_t[r_{t+1}] - r^F_{t+1} = \mathbb{E}_t\left[r^C_{t+1}\{r_{t+1} - r^F_{t+1}\}\right]$$
$$\Rightarrow \mathbb{E}_t[r_{t+1}] - r^F_{t+1} = Cov_t(r^C_{t+1}, r_{t+1} - r^F_{t+1}) + \mathbb{E}_t\left[r^C_{t+1}\right]\mathbb{E}_t\left[r_{t+1} - r^F_{t+1}\right]$$

where the last line uses our trick from last class $\mathbb{E}[xy] = Cov(x, y) + \mathbb{E}[x]\mathbb{E}[y]$.

Derivation

- Collecting terms then gives

$$\mathbb{E}_t[r_{t+1}] - r^F_{t+1} = \frac{1}{1 - \mathbb{E}_t[r^C_{t+1}]} Cov_t(r^C_{t+1}, r_{t+1} - r^F_{t+1}) \tag{7}$$

- See that (7) must hold for all assets in the economy.

- Then it must also hold for the asset delivering r^C_{t+1}.

- Then

$$\begin{aligned}\mathbb{E}_t[r^C_{t+1}] - r^F_{t+1} &= \frac{1}{1 - \mathbb{E}_t[r^C_{t+1}]} Cov_t(r^C_{t+1}, r^C_{t+1} - r^F_{t+1}) \\ &= \frac{1}{1 - \mathbb{E}_t[r^C_{t+1}]} Var_t(r^C_{t+1}) \end{aligned}\tag{8}$$

Derivation

- Then divide (7) by (8) to get

$$\frac{\mathbb{E}_t[r_{t+1}] - r^F_{t+1}}{\mathbb{E}_t[r^C_{t+1}] - r^F_{t+1}} = \frac{Cov_t(r^C_{t+1}, r_{t+1} - r^F_{t+1})}{Var_t(r^C_{t+1})}$$

$$\Rightarrow \mathbb{E}_t[r_{t+1}] = r^F_{t+1} + \frac{Cov_t(r^C_{t+1}, r_{t+1} - r^F_{t+1})}{Var_t(r^C_{t+1})}\{\mathbb{E}_t[r^C_{t+1}] - r^F_{t+1}\}$$

$$= r^F_{t+1} + \beta^C\{\mathbb{E}_t[r^C_{t+1}] - r^F_{t+1}\}$$

where $\beta^C \equiv \frac{Cov_t(r^C_{t+1}, r_{t+1} - r^F_{t+1})}{Var_t(r^C_{t+1})}$.

Derivation

- Re-written from the last slide

$$\mathbb{E}_t[r_{t+1}] = r^F_{t+1} + \beta^C\{\mathbb{E}_t[r^C_{t+1}] - r^F_{t+1}\}$$

- We have a micro-founded regression equation for returns

$$r_{t+1} = r^F_{t+1} + \beta^C\{r^C_{t+1} - r^F_{t+1}\} + u_{t+1}$$

- If we have data on asset returns and aggregate consumption, we can run this regression to get an estimate for β^C.

This is a Big Deal

- This is remarkable...why?
- It says that we can get an estimate of the excess return on an individual asset, just by regressing against an aggregate variable.
- A dissertation discussion with a student in this class helped me to understand this point.
- No need to think about firm-level information when forecasting its excess return.

Risk

- What's the intuition for this regression equation?
- It says that assets whose return is positively correlated with the excess return on the consumption asset have higher expected returns.
- Risk and return!
- A riskier asset has a return that moves more with the market.

Taking the model to the data

- We've studied the consumption CAPM.
- An older idea in finance is CAPM (regular CAPM).
- It uses a slightly different model doing similar derivations to what we've done here, (but a lot more painful in my opinion).
- CAPM says that expected returns are given by

$$\mathbb{E}_t[r_{t+1}] = r^F_{t+1} + \beta^M\{\mathbb{E}_t[r^M_{t+1}] - r^F_{t+1}\}$$

where r^M_{t+1} is the return on the market portfolio.

- You can think of the market portfolio as something like the S&P500 (or FTSE here in Britain).

Taking the model to the data

- Theoretically, consumption CAPM and CAPM can be the same under certain assumptions.
- Empirically, which is a better predictor of returns?
- CAPM:

$$\mathbb{E}_t[r_{t+1}] = r^F_{t+1} + \beta^M\{\mathbb{E}_t[r^M_{t+1}] - r^F_{t+1}\}$$

- Or consumption CAPM

$$\mathbb{E}_t[r_{t+1}] = r^F_{t+1} + \beta^C\{\mathbb{E}_t[r^C_{t+1}] - r^F_{t+1}\}$$

- Studies have shown that CAPM fits the data better.

Taking the model to the data

- So the typical regression people run in the empirical literature is

$$r_{t+1} = r^F_{t+1} + \beta^M \{ r^M_{t+1} - r^F_{t+1} \} + u_{t+1}$$

- If the theory is right, regressing asset returns against the riskless rate and the excess return of the market portfolio should yield unbiased estimates of β^M.

- Testing the theory: does adding extra regressors to the right-side change the estimate of β^M?

- Does the theory exclude important variables?

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Adding more regressors

- Multi-factor models keep the CAPM framework but add additional variables to the right-side.
- In its general form

$$r_{t+1} = r^F_{t+1} + \beta_1 f_{1,t+1} + \beta_2 f_{2,t+1} + \beta_3 f_{3,t+1} + \ldots + \beta_N f_{N,t+1}$$

where $f_{1,t+1} = r^M_{t+1} - r^F_{t+1}$ and $\beta_1 = \beta^M$.

Adding more regressors

- Adding more regressors can increase the explanatory power of the regression.
- Remove any potential bias from estimates of β^M.
- Just a statistical model though: no theory is driving what additional factors we need to include!

Fama & French (1992): 3 Factors

- Ran the CAPM regressions in the cross-section and found that it didn't work for the U.S. stock market.
- Found that another two factors had strong predictive power:
  (1) SMB: small (market capitalisation) minus big.
  (2) HML: high (book-to-market ratio) minus low.
- Motivated by the observation that small market cap firms tend to out-perform those with large market cap and similarly for high market to book ratio firms relative to low.

Fama & French (1992): 3 Factors

- Claim that these are proxies for macro factors.
- SMB captures the historical excess return of small size over big size firms [size risk].
- HML captures historical premium for “value” stocks over “growth” stocks [value risk].
- Smaller firms are riskier, so you’d expect them to fetch a higher return.
- Higher book to market: more capital than future profitable projects and therefore riskier.

Fama & French (1992): 3 Factors

- Regression takes the form

$$r_{t+1} = r^F_{t+1} + \beta^M [r^M_{t+1} - r^F_{t+1}] + \beta^{SMB} r^{SMB}_{t+1} + \beta^{HML} r^{HML}_{t+1}$$

  where r^{SMB}_{t+1} and r^{HML}_{t+1} are the size premium and value premium respectively.

- You can download data series for these variables from French's website.

- Their regressions give adjusted R squared of around 90% when explaining returns on portfolios of stocks.

Carhart (1997): 4 Factors

- Includes an additional factor to capture momentum.
- Momentum: rising prices keep rising, falling prices keep falling.
- Include a regressor that looks at lagged premium of "winning" firms' returns over "loosing" firms' returns.

Fama & French (2015): 5 Factors

- Additional two factors to account for profitability and investment.

- Five factor regression takes the form

$$r_{t+1} = r^F_{t+1} + \beta^M [r^M_{t+1} - r^F_{t+1}] + \beta^{SMB} r^{SMB}_{t+1} + \beta^{HML} r^{HML}_{t+1} + \beta^{RMW} r^{RMW}_{t+1} + \beta^{CMA} r^{CMA}_{t+1}$$

where r^{RMW}_{t+1} is the difference between the returns on diversified portfolios of stocks with robust and weak profitability and r^{CMA}_{t+1} is the difference between the returns on diversified portfolios of the stocks of low and high investment firms.

- Foye (2017): tested the model on the UK...didn't do well.

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Summary

- These methods are used in industry quite a bit.
- Simple to implement.
- But again, no theory beyond the market risk premium factor.