Lecture 14: New Keynesian Model Part VI
Zero Lower Bound

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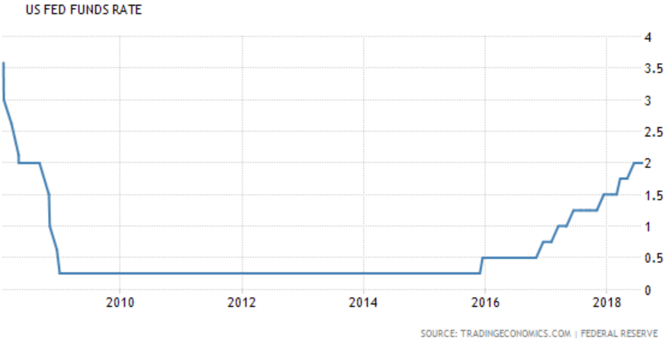
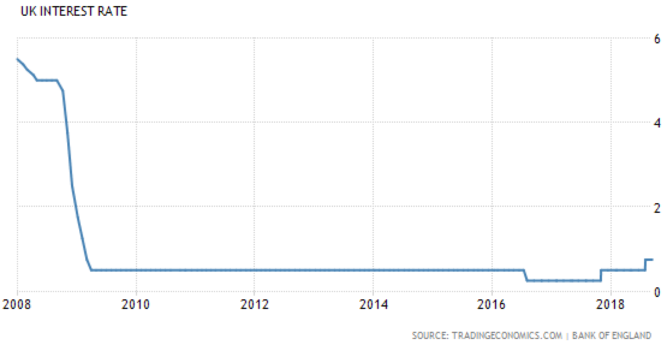
Roadmap

Motivation

- How effective is fiscal stimulus when the nominal interest rate approaches zero?



Motivation


UK INTEREST RATE
6
4
2
0
2008
2010
2012
2014
2016
2018
SOURCE: TRADINGECONOMICS.COM | BANK OF ENGLAND

Motivation

- Woodford "Simple analytics of the government expenditure multiplier" AEJ: Macroeconomics, 2011.
- The setup here might seem strange at first. Have a look at the paper for more explanation if you need it.

Today

- We will proceed in three steps:
  (1) Adjust the interest rate policy rule to account for the lower bound.
  (2) Study when the constraint binds or is slack.
    - "Run" a simulation experiment with a crisis and government intervention at the bound.

Roadmap

Setup

- We assume that there is some exogenous spread that affects the interest rate faced by the households in the model.
- If the policy rate is still i_t then household faces rate

$$i_t + \Delta_t$$

where Δ_t is a spread that follows an exogenous process.

Setup

- Think of Δ_t as a reduced-form way of capturing the health of financial markets.
- Assume that it can take two values:
  (1) $\Delta_H = 0$, which resembles regular times.
  (2) $\Delta_L > 0$, which resembles crisis times.

Credit Spreads during Crisis Times

Policy Rule

- We'll assume that the central bank follows the following rule

$$I_t = \max\{1, I_t^*\} \tag{1}$$

$$I_t^* = \bar{I}^* \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} \tag{2}$$

where I_t is the gross nominal interest rate and Π_t is the gross nominal inflation rate.

- Notice that this is comprised of two parts: a target rule and the zero lower bound.

- Why is there a one in the first entry rather than a zero?

- I exclude output gap from the target rule for simplicity.

Policy Rule

- It might not be obvious at first, but the target policy rule in (1) should look familiar.
- Consider log-linearising it:

$$\hat{i}_t^* = \phi_\pi \hat{\pi}_t$$

which is just our standard interest rate rule without the output gap!

Roadmap

Binding ZLB

- Follows standard target rule when the constraint doesn't bind.
- When does it bind?

$$
\begin{aligned}
I_t^* &< 1 \\
\Rightarrow \bar{I}^* \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} &< 1 \\
\Rightarrow \Pi_t &< \bar{\Pi}\left(\bar{I}^*\right)^{-\frac{1}{\phi_\pi}}.
\end{aligned}
$$

- Take $\bar{\Pi} = 1$ (i.e. the "normal" zero inflation steady state).
- Then follows from the steady state Euler equation that $\bar{I}^* = \beta^{-1}$.

Binding ZLB

- Then the ZLB binds when

$$\Pi_t < \beta^{\frac{1}{\phi_\pi}}$$

  where $\beta^{\frac{1}{\phi_\pi}} < 1$ given $\beta < 1$.

- Binds in periods of deflation of a specific magnitude.
- If deflation gets too large in magnitude, then the optimal nominal rate implied by the target rule will be negative.
- Monetary policy becomes ineffective.

Binding ZLB

- We'll write the constraint in semi-log linear form as

$$i_t = \max\{0, -\log(\beta) + \phi_\pi \hat{\pi}_t\}$$

where $i_t = \log(I_t)$ is the net nominal interest rate (approximately).

- Why can't I just use $\hat{i}_t = \max(0, \phi_\pi \hat{\pi}_t)$?

Roadmap

Experiment Design: Credit Spreads

- Simulate a crisis where we start in the “low” state with $\Delta = \Delta_L > 0$.
- Assume that there is a transition probability given by

$$
\begin{aligned}
\mathbb{P}\mathrm{r}[\Delta_{t+1} = \Delta_L | \Delta_t = \Delta_L] &= \mu \\
\mathbb{P}\mathrm{r}[\Delta_{t+1} = \Delta_H | \Delta_t = \Delta_L] &= 1 - \mu \\
\mathbb{P}\mathrm{r}[\Delta_{t+1} = \Delta_H | \Delta_t = \Delta_H] &= 1
\end{aligned}
$$

  meaning that there is a probability μ that we'll come out of the crisis but once we're out of it, we're out of it for good.

- Economy will state in the crisis state for some random duration T.

Experiment Design: Fiscal Policy

- The fiscal authority has some "normal-time" level of purchases given by $\bar{G}$.
- In the crisis state, they expand their activity to some $G_L > \bar{G}$ until the economy returns to the "normal" state.

Period T onwards

- After the economy leaves the crisis state, we go back to the "normal" steady state.
- From t onwards, the deviations of variables from steady state will be zero.
- I.e. $\Delta = 0, \hat{\pi}_H = 0, \hat{y}_H^g = 0, i_H = -\log(\beta)$ are the values from T onwards.
- Why is there no transition period?

Before Period T: new Keynesian Phillips Curve

- The endogenous variables will all assume values associated with the value Δ_L.
- I.e. want to solve for $\hat{\pi}_L, \hat{y}_L^g, i_L$ etc.
- See from the new Keynesian Phillips curve that

$$\begin{aligned} \hat{\pi}_L &= \beta \left\{ \mu \hat{\pi}_L + (1-\mu)\hat{\pi}_H \right\} + \kappa \hat{y}_L^g \\ &= \beta \left\{ \mu \hat{\pi}_L \right\} + \kappa \hat{y}_L^g \\ \Rightarrow \hat{\pi}_L &= \frac{\kappa}{1-\mu\beta} \hat{y}_L^g \end{aligned}$$

Before Period T: Natural Rate of Interest

- See that the natural rate of interest is given by

$$r_t^n = -\log(\beta) - \sigma(1-\Gamma)\mathbb{E}_t[\hat{g}_{t+1} - \hat{g}_t]$$

how does this differ from last class?

- Under our assumptions regarding the crisis/normal times, this comes down to

$$\begin{aligned} r_t^n &= -\log(\beta) - \sigma(1-\Gamma)\mathbb{E}_t[\mu(\hat{g}_L - \hat{g}_L) + (1-\mu)(\hat{g}_H - \hat{g}_L)] \\ &= -\log(\beta) + \sigma(1-\Gamma)(1-\mu)(\hat{g}_L) \end{aligned}$$

Before Period T: Dynamic IS Curve

- Dynamic IS curve given by

$$\hat{y}^g_L = \mathbb{E}_t[\hat{y}^g_{t+1}] - \frac{1}{\tilde{\sigma}}[i_t - \mathbb{E}_t\hat{\pi}_{t+1} - r^n_t]$$

how does this differ from last class?

- Now plug in the natural rate of interest to get

$$\hat{y}^g_L = \mu\hat{y}^g_L - \frac{1}{\tilde{\sigma}}[i_L + \Delta_L - \mu\hat{\pi}_L - r^n_L]$$

Before Period T: Dynamic IS Curve

- Now let $r_L = -\log(\beta) - \Delta_L$. Then we can write

$$
\begin{aligned}
(1-\mu)\hat{y}_L^g &= \frac{1}{\tilde{\sigma}}[r_L - i_L] + \frac{\mu\kappa}{\tilde{\sigma}(1-\mu\beta)}\hat{y}_L^g + (1-\mu)(1-\Gamma)\hat{g}_L \\
\Rightarrow \hat{y}_L^g &= \zeta_r(r_L - i_L) + \zeta_g\hat{g}_L
\end{aligned}
$$

where

$$
\begin{aligned}
\zeta_r &= \frac{(1-\mu\beta)}{\tilde{\sigma}(1-\mu)(1-\mu\beta) - \mu\kappa} \\
\zeta_g &= \frac{\tilde{\sigma}(1-\mu)(1-\mu\beta)}{\tilde{\sigma}(1-\mu)(1-\mu\beta) - \mu\kappa}(1-\Gamma)
\end{aligned}
$$

- We're not done yet though, as i_L is endogenous.

Back to the Nominal Rate

- Substitute the expression for the output gap into the monetary policy rule

$$
\begin{aligned}
i_L &= \max\{0, -\log(\beta) + \phi_\pi \hat{\pi}_t\} \\
&= \max\left\{0, -\log(\beta) + \phi_\pi \frac{\kappa}{1-\mu\beta}[\zeta_r(r_L - i_L) + \zeta_g \hat{g}_L]\right\}
\end{aligned}
$$

where see that everything is exogenous inside the max operator except for i_L and $\hat{g}_L$.

Back to the Nominal Rate: No Fiscal Stimulus

- If there is no fiscal stimulus in crisis, then $\hat{g}_L = 0$.
- ZLB binds when

$$-\log(\beta) + \phi_\pi \frac{\kappa}{1-\mu\beta}\zeta_r r_L < 0$$

$$\Rightarrow r_L < \log(\beta)\frac{1}{\phi_\pi \frac{\kappa}{1-\mu\beta}\zeta_r} \equiv r^* < 0$$

meaning that Δ_L needs to be sufficiently large as $r_L = -\log(\beta) - \Delta_L$.

- In the absence of fiscal stimulus, a really big crisis will trigger the ZLB.

Back to the Nominal Rate:...OK Maybe a Little Stimulus...

- Notice that if $\hat{g}_L$ is sufficiently small then the ZLB will still bind assuming $r_L < r^*$.

- That is — we need for

$$\hat{g}_L < \hat{g}_L^* \equiv -\frac{-\log(\beta) + \phi_\pi \frac{\kappa}{1-\mu\beta}\zeta_r r_L}{\phi_\pi \frac{\kappa}{1-\mu\beta}\zeta_g}$$

where the object on the right is positive due to the restriction on r_L.

Multiplier

- Then see that, due to our restrictions:

$$\hat{y}_t^g = \zeta_r r_L + \zeta_g \hat{g}_L < 0$$

where the inequality follows from $r_L < 0$ and $\hat{g}_L$ being sufficiently small.

- Follows then that the multiplier is given by

$$\begin{aligned}\frac{\partial \hat{y}_L}{\partial \hat{g}_L} &= \frac{\partial \hat{y}_L^g}{\partial \hat{g}_L} + \frac{\partial \hat{y}_L^n}{\partial \hat{g}_L} \\ &= \zeta_g + \Gamma\end{aligned}$$

where the first equality follows from the definition of the output gap.

Multiplier

- Recall that

$$\zeta_g = \frac{\tilde{\sigma}(1-\mu)(1-\mu\beta)}{\tilde{\sigma}(1-\mu)(1-\mu\beta)-\mu\kappa}(1-\Gamma)$$
$$> (1-\Gamma)$$

  which means that the multiplier is larger than one at the ZLB below the critical threshold for $\hat{g}_L$.

- Notice also that the inflation rate increases with the fiscal stimulus.

Roadmap

Takeaways

- When the ZLB binds, it gives scope for fiscal stimulus to be more effective.
- Fiscal multipliers are larger than one for stimulus below a certain threshold.
- No response of monetary policy.