Lecture 13: New Keynesian Model Part V
Fiscal Multipliers

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Roadmap

1. Introduction
2. Flexible Price Multiplier
3. Sticky Price Multiplier
4. Comparison and Conclusion

Motivation

- During times of economic turmoil, often governments seek to boost activity through spending programs.
- Dates-back to the old ideas of Keynes.
- If the fiscal authority increases their expenditures by x%, how much will output increase by?
- How "effective" is fiscal policy in the economy as a stimulating tool.

Motivation

- This lecture will proceed in two steps.
- Study the impact of fiscal policy on
  (1) Flexible price equilibrium.
  (2) New Keynesian equilibrium.

Model Environment with Government Spending

- In this lecture, we'll assume that there are no productivity or monetary policy shocks.
- The only stochastic variable is government spending, G_t.
- Log-linearised version denoted by $\hat{g}_t$.
- I'll assume that there is one single process for G_t.
- All other variables will either have no superscript, (if sticky price model), or a superscript n if flexible price variable.

Roadmap

Household Problem

- Problem:

$$\max_{\{C_t^n, N_t^n, B_{t+1}^n\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^n)^{1-\sigma}}{1-\sigma} - \frac{(N_t^n)^{1+\psi}}{1+\psi} \right]$$

subject to their budget constraint

$$P_t^n C_t^n + Q_t^n B_{t+1}^n \leqslant W_t^n B_t^n + B_t^n + D_t^n - V_t^n$$

- where V_t is a lump-sum tax that is levied by the government on the households.
- Recall that B_t^n are government bonds.

Household's Problem: Optimality

- Lagrangian given by

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^n)1-\sigma}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} \right] + \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_t^n \left[W_t^n N_t^n + B_t^n - V_t^n - Q_t^n B_{t+1}^n - P_t^n C_t^n \right]$$

Household Optimality: First Order Conditions

- FOCs:

$$\frac{\partial \mathcal{L}}{\partial C_t^n} = 0 \Rightarrow \beta^t (C_t^n)^{-\sigma} - P_t^n \lambda_t^n = 0 \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial N_t^n} = 0 \Rightarrow -\beta^t (N_t^n)^{\psi} + \lambda_t^n W_t^n = 0 \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}^n} = 0 \Rightarrow -Q_t^n \lambda_t^n + \mathbb{E}_t[\lambda_{t+1}^n] = 0 \tag{3}$$

- Notice that nothing changes here relative to the case without government spending.

Fiscal Authority

- We'll assume that their behaviour is exogenous.
- Fiscal policy rule of the form

$$\log(G_t) = \rho_G log(G_{t-1}) + \epsilon_{G,t}$$

for $0 < \rho_G < 1$.

- Government budget constraint

$$B_t^n + P_t^n G_t = V_t^n + q_t^n B_{t+1}^n.$$

- Sources of funds: new borrowing from households and taxes.
- Uses of funds: repaying old borrowing and government spending.

Natural Rate of Interest

- An exogenous rate that comes from the linearised household Euler equation

$$\hat{c}_t^n = \mathbb{E}_t[\hat{c}_{t+1}^n] - \frac{1}{\sigma}\left[\hat{i}_t^n - \mathbb{E}_t\hat{\pi}_{t+1}^n\right]$$
$$\Rightarrow \hat{r}_t^n = \sigma\mathbb{E}_t[\hat{c}_{t+1}^n - \hat{c}_t^n]$$

Resource Constraint

- Government purchases now enter into the constraint

$$C_t^n + G_t = Y_t^n$$

Linearised System

- Household block

$$\varphi \hat{n}_t^n + \sigma \hat{c}_t^n = \hat{w}_t^n - \hat{p}_t^n \quad (4)$$
$$\hat{r}_t^n = \sigma \mathbb{E}_t[\hat{c}_{t+1} - \hat{c}_t^n] \quad (5)$$

- Firm block

$$\hat{y}_t^n - \hat{n}_t^n = \hat{w}_t^n - \hat{p}_t^n \quad (6)$$
$$\hat{y}_t^n = (1-\alpha)\hat{n}_t^n \quad (7)$$

- Government block

$$\hat{g}_t = \rho_G \hat{g}_{t-1} + \epsilon_{G,t} \quad (8)$$

- Market clearing

$$\hat{y}_t^n = \frac{\bar{C}}{\bar{Y}} \hat{c}_t^n + \frac{\bar{G}}{\bar{Y}} \hat{g}_t \quad (9)$$

- Six equations in seven unknowns. What's the issue?

Fiscal Multiplier

- Set $\hat{p}_t^n = 0$ as the numeraire.
- Substitute (4) into (6) to get

$$\begin{aligned} \varphi \hat{n}_t^n + \sigma \hat{c}_t^n &= \hat{y}_t^n - \hat{n}_t^n \\ \Rightarrow \hat{y}_t^n &= (1+\varphi)\hat{n}_t^n + \sigma \hat{c}_t^n \end{aligned} \tag{10}$$

Substitute (7) into (10) to get

$$\begin{aligned} \hat{y}_t^n &= \frac{1+\varphi}{1-\alpha}\hat{y}_t^n + \sigma \hat{c}_t^n \\ \Rightarrow \hat{y}_t^n &= -\left(\frac{1-\alpha}{\varphi+\sigma}\right)\sigma \hat{c}_t^n \end{aligned} \tag{11}$$

Fiscal Multiplier

- Then substitute (9) into (11) and re-arrange to get

$$\hat{y}_t^n = \frac{\sigma \frac{\bar{G}}{\bar{C}}}{\frac{\varphi+\sigma}{1-\alpha} + \frac{\bar{Y}}{\bar{G}}\sigma}\hat{g}_t$$

what is the coefficient?

- See that

$$\frac{\partial \hat{y}_t^n}{\partial \hat{g}_t} = \frac{\sigma \frac{\bar{G}}{\bar{C}}}{\frac{\varphi+\sigma}{1-\alpha} + \frac{\bar{Y}}{\bar{G}}\sigma}$$

...this is an elasticity!

Fiscal Multiplier

- What can we say about its size?
- See that

$$\frac{\bar{G}}{\bar{C}} = \frac{\bar{G}}{\bar{Y}}\frac{\bar{Y}}{\bar{C}}$$
$$= \frac{\bar{G}}{\bar{Y}}\left[1 - \frac{\bar{G}}{\bar{Y}}\right]^{-1}$$
$$\Rightarrow \frac{\sigma\frac{\bar{G}}{\bar{C}}}{\frac{\varphi+\sigma}{1-\alpha} + \frac{\bar{Y}}{\bar{G}}\sigma} = \frac{1}{\left[\frac{\bar{Y}}{\bar{G}} - 1\right]\left\{\frac{\varphi+\sigma}{\sigma(1-\alpha)} + \frac{\bar{Y}}{\bar{G}}\right\}} < 1,$$

where the inequality follows from the fact that $\frac{\bar{Y}}{\bar{G}} > 1$.

Fiscal Multiplier

- So it follows that

$$\frac{\partial \hat{y}_t^n}{\partial \hat{g}_t} < 1$$

meaning that a 1% increase in G_t will lead to a less than proportional increase in Y_t^n.

Roadmap

Motivation

- Now let's think about the model with sticky prices.
- Household FOCs will still be the same, (obviously without the n superscripts though).
- As will the resource constraint.
- What will the new Keynesian Phillips curve and dynamic IS curve look like?

Dynamic IS Curve

- What does the output gap look like?
- We've already characterised the flexible price equilibrium earlier.
- See that

$$\begin{aligned}\hat{y}_t^g &= \hat{y}_t - \hat{y}_t^n \\ &= \frac{\bar{C}}{\bar{Y}}\hat{c}_t + \frac{\bar{G}}{\bar{Y}}\hat{g}_t - \frac{\sigma\frac{\bar{G}}{\bar{C}}}{\frac{\varphi+\sigma}{1-\alpha} + \frac{\bar{Y}}{\bar{G}}\sigma}\hat{g}_t \\ &= \frac{\bar{C}}{\bar{Y}}\hat{c}_t + \Gamma\hat{g}_t\end{aligned} \tag{12}$$

where $\Gamma = \left\{\frac{\bar{G}}{\bar{Y}} - \frac{\sigma\frac{\bar{G}}{\bar{C}}}{\frac{\varphi+\sigma}{1-\alpha} + \frac{\bar{Y}}{\bar{G}}\sigma}\right\}$.

Dynamic IS Curve

- Now plug equation (12) into the household's Euler equation to get

$$\hat{c}_t = \mathbb{E}_t(\hat{c}_{t+1}) - \frac{1}{\sigma}[\hat{i}_t - \mathbb{E}_t(\hat{\pi}_{t+1})]$$

$$\Rightarrow \hat{y}_t^g - \Gamma\hat{g}_t = \mathbb{E}_t[\hat{y}_{t+1}^g] - \Gamma\mathbb{E}_t\hat{g}_{t+1} - \frac{1}{\sigma}\frac{\bar{C}}{\bar{Y}}[\hat{i}_t - \mathbb{E}_t(\hat{\pi}_{t+1})]$$

$$\Rightarrow \hat{y}_t^g = \mathbb{E}_t[\hat{y}_{t+1}^g] - \frac{1}{\tilde{\sigma}}[\hat{i}_t - \mathbb{E}_t(\hat{\pi}_{t+1}) - \hat{r}_t^n]$$

where $\hat{r}_t^n = \tilde{\sigma}\frac{\bar{Y}}{\bar{C}}(1-\Gamma)(1-\rho_G)\hat{g}_t$ and $\frac{1}{\tilde{\sigma}} = \frac{\bar{C}}{\bar{Y}}\frac{1}{\sigma}$.

- The natural rate of interest now depends on the fiscal policy shock.

- It didn't depend on the monetary policy shock though. What's going on?

New Keynesian Phillips Curve

- Does this change?
- No: the firms are just taking demand as exogenous anyway.

Linearised System

$$\hat{y}_t^g = \mathbb{E}_t[\hat{y}_{t+1}^g] - \frac{1}{\tilde{\sigma}}[\hat{i}_t - \mathbb{E}_t(\hat{\pi}_{t+1}) - \hat{r}_t^n]$$

$$\hat{\pi}_t = \beta\mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa\hat{y}_t^g$$

$$\hat{i}_t = \phi_\pi\hat{\pi}_t + \phi_y\hat{y}_t^g$$

$$\hat{r}_t^n = \tilde{\sigma}\frac{\bar{Y}}{\bar{C}}(1-\Gamma)(1-\rho_G)\hat{g}_t$$

$$\hat{g}_t = \rho_G\hat{g}_{t-1} + \epsilon_{g,t}$$

Undetermined Coefficients

- Can we get any further?
- Conjecture that

$$\hat{\pi}_t = \gamma_{\pi g} \hat{g}_t$$
$$\hat{y}_t^g = \gamma_{yg} \hat{g}_t$$

and utilise method of undetermined coefficients.

Undetermined Coefficients

- See that the new Keynesian Phillips curve implies that

$$
\begin{aligned}
\gamma_{\pi g}\hat{g}_t &= \beta\rho_G\gamma_{\pi g}\hat{g}_t + \kappa\gamma_{yg}\hat{g}_t \\
\Rightarrow \gamma_{\pi g} - \beta\rho_G\gamma_{\pi g} - \kappa\gamma_{yg} &= 0 \\
\Rightarrow \gamma_{\pi g} &= \frac{\kappa}{1-\beta\rho_G}\gamma_{yg}
\end{aligned}
$$

Undetermined Coefficients

- Then the dynamic IS curve gives that (exercise: check)

$$\gamma_{yg} = \frac{\tilde{\sigma}\frac{\bar{Y}}{\bar{C}}(1-\Gamma)(1-\rho_G)\hat{g}_t}{\tilde{\sigma}(1-\rho_G)+\phi_y-(\rho_G-\phi_\pi)\frac{\kappa}{1-\beta\rho_G}}$$

Fiscal Multiplier

- We can then think about what happens to the output gap with a fiscal shock.

$$\hat{y}_t^g = \frac{\tilde{\sigma}\frac{\bar{Y}}{\bar{C}}(1-\Gamma)(1-\rho_G)}{\tilde{\sigma}(1-\rho_G)+\phi_y+(\phi_\pi-\rho_G)\frac{\kappa}{1-\beta\rho_G}}\hat{g}_t$$

where recall $\Gamma = \left\{\frac{\bar{G}}{\bar{Y}} - \frac{\sigma\frac{\bar{G}}{\bar{C}}}{\frac{\varphi+\sigma}{1-\alpha}+\frac{\bar{Y}}{\bar{G}}\sigma}\right\}$.

- Recall that we also showed in the previous section that

$$\frac{\sigma\frac{\bar{G}}{\bar{C}}}{\frac{\varphi+\sigma}{1-\alpha}+\frac{\bar{Y}}{\bar{G}}\sigma} < 1$$

$$\Rightarrow \Gamma = \frac{\bar{G}}{\bar{Y}} - \frac{\sigma\frac{\bar{G}}{\bar{C}}}{\frac{\varphi+\sigma}{1-\alpha}+\frac{\bar{Y}}{\bar{G}}\sigma} < 1,$$

which means that the output gap increases with positive fiscal shocks.

Fiscal Multiplier

- Then gives the multiplier on output

$$\begin{aligned}\hat{y}_t &= \hat{y}_t^g + \hat{y}_t^n \\ &= \left\{ \frac{\sigma(1-\Gamma)(1-\rho_G)}{\sigma \frac{\bar{Y}}{\bar{C}}(1-\rho_G) + \phi_y + (\phi_\pi - \rho_G)\frac{\kappa}{1-\beta\rho_G}} + \Gamma \right\} \hat{g}_t\end{aligned}$$

which is clearly bigger than that under the flexible price equilibrium.

Fiscal Multiplier

- Notice that it's also less than one though as

$$\frac{\sigma(1-\Gamma)(1-\rho_G)}{\sigma\frac{\bar{Y}}{\bar{C}}(1-\rho_G)+\phi_y+(\phi_\pi-\rho_G)\frac{\kappa}{1-\beta\rho_G}} =$$

$$\frac{(1-\Gamma)}{\frac{\bar{Y}}{\bar{C}}+\frac{\phi_y}{\sigma(1-\rho_G)}+(\phi_\pi-\rho_G)\frac{\kappa}{(1-\beta\rho_G)\sigma(1-\rho_G)}} < 1-\Gamma$$

where the inequality follows from the fact that $\bar{Y}/\bar{C} > 1$ and $\phi_\pi > 1 > \rho_G$.

Roadmap

Flexible v.s. Sticky Prices

- Multiplier is larger under sticky prices than flexible.
- Both still less than one though.
- Due to the reaction of the monetary policy rule.
- Increasing in the degree of price stickiness (higher θ reduces κ).
- Decreasing in monetary policy reactiveness.

Next Class

- Monetary policy accommodation.
- Increases the size of the multiplier: can be above one.