Lecture 12: New Keynesian Model Part IV
Optimal Monetary Policy

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Roadmap

Motivation

- Two main sources of distortion in the new Keynesian model.
  - **(1)** Markups due to imperfect competition.
  - **(2)** Price dispersion due to nominal rigidities.
- Notice that (1) is present in the flexible price equilibrium.
- Affects the natural level of output and interest ("long-run" distortion).
- We can interpret (2) as a "short-run" distortion.
- Which of the two distortions should central bankers be concerned about? Why?
- Assume that (1) is solved by a fiscal authority (see exercise set) and focus on monetary authority's problem.

Motivation

- Recall in earlier lectures, we assumed that monetary policy just followed a rule of the form

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t^g + \nu_t.$$

- This was something that we just took as exogenous.
- We're now going to think about how $\hat{i}_t$ should be set optimally.

Roadmap

Setup

- What objectives do central banks typically have?
- Targeting inflation.
- Stabilising business cycles (referred to as a "dual mandate").
- In our model, this means keeping $\hat{\pi}_t$ and $\hat{y}_t^g$ small.

Reduced-Form Objective

- Assume that the monetary authority has discretion over the nominal interest rate $\hat{i}_t$.
- Seeks to minimise a "loss function", which is increasing in the output gap and inflation.
- Assume it takes the form

$$\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\{\hat{\pi}_t\}^2 + \omega\{\hat{y}_t^g\}^2\right) \tag{1}$$

where $\omega > 0$ is the weight it puts on keeping the economy stable.

- Why do we use this functional form? Why does the number 2 make so many appearances?

Microfoundations

- If we were to truly micro-found the objective of the monetary authority, what would it look like?
- Household welfare!
- We can show that this reduced-form objective in equation (1) is equivalent to household objective up to an approximation.
- I don't expect you to show this though, (it's just uninformative algebra).

Policy Tools

- In principle, there are two ways the central bank could make its choice of the sequence $\{\hat{i}_t\}_{t=0}^{\infty}$.
  - **(a)** Choose the whole sequence at $t = 0$: commitment.
  - **(b)** Choose i_t on a period-by-period basis: discretion.
- In this class, we'll focus on the discretion case.

Discretion

- Recall our system of equations (excluding monetary policy) was given by

$$\hat{\pi}_t = \kappa \hat{y}_t^g + \beta \mathbb{E}_t[\hat{\pi}_{t+1}]$$
$$\hat{y}_t^g = \mathbb{E}_t[\hat{y}_{t+1}^g] - \frac{1}{\sigma}(\hat{i}_t - \mathbb{E}_t[\hat{\pi}_{t+1}] - \hat{r}_t^n)$$

  in addition to the exogenous processes.

- We call $\hat{r}_t^n$ the natural rate of interest as it's the real rate that would prevail in the flexible price equilibrium.

- It's exogenous and just depends on productivity $\hat{a}_t$ (see L10).

- See then that $\hat{i}_t, \hat{r}_t^n \Rightarrow \hat{\pi}_t, \hat{y}_t^g$.

Discretion

- Can then think of central bank's problem as being

$$\min_{\hat{\pi}_t, \hat{y}_t^g} \frac{1}{2} \left(\{\hat{\pi}_t\}^2 + \omega \{\hat{y}_t^g\}^2 \right) \tag{2}$$

subject to

$$\hat{\pi}_t = \kappa \hat{y}_t^g + \beta \mathbb{E}_t[\hat{\pi}_{t+1}]$$

- We're now thinking about the new Keynesian Phillips curve as a constraint.

- The sum in the objective drops-out given that the problem is being solved each period (discretion).

Roadmap

Lagrangian

- Lagrangian given by

$$\mathcal{L} = \frac{1}{2}\left(\{\hat{\pi}_t\}^2 + \omega\{\hat{y}_t^g\}^2\right) + \lambda(\hat{\pi}_t - \kappa\hat{y}_t^g - \beta\mathbb{E}_t[\hat{\pi}_{t+1}])$$

Optimality Conditions

- FOCs given by

$$\frac{\partial \mathcal{L}}{\partial \hat{\pi}_t} = 0 \Rightarrow \hat{\pi}_t = \lambda$$
$$\frac{\partial \mathcal{L}}{\partial \hat{y}_t^g} = 0 \Rightarrow \omega \hat{y}_t^g = -\lambda\kappa$$

- We can combine these to yield

$$\hat{\pi}_t = -\frac{\omega}{\kappa}\hat{y}_t^g \tag{3}$$

- Says that when the output gap is positive, central bank seeks to reduce inflation (and vice-versa).

Optimality Conditions

- The global minimum to the objective function is

$$\hat{\pi}_t = \hat{y}_t^g = 0$$

which also satisfies FOC (3).

- Since the central bank re-optimises each period, then they'll always pick the same solution.

- Means that

$$\mathbb{E}_t[\hat{\pi}_t] = \mathbb{E}_t[\hat{y}_t^g] = 0$$

which satisfies the Phillips curve constraint.

Optimality Conditions

- What does this mean then in terms of $\hat{i}_t$?
- See from the dynamic IS curve that

$$\hat{i}_t = \hat{r}_t^n \tag{4}$$

  meaning that the nominal rate should be set equal to the real natural rate.

- Note also that $\hat{i}_t$ here is also the real interest rate since expected inflation is zero.

Roadmap

Modified Taylor Rule

- How can the central bank achieve

$$\hat{i}_t = \hat{r}_t^n? \tag{5}$$

- Could they just employ (5) as their monetary policy rule?
- Answer turns out to be no for non-obvious reasons.

Stability of Dynamic Systems: Scalars

- Say that we want a stable solution to an equation of the form

$$x_{t+1} = \rho x_t.$$

where $x_t, \rho \in \mathbb{R}$.

- We can either solve this forwards or backwards.
- To solve backwards, we need an initial condition.
- Stable solution when solving backwards if

$$|\rho| < 1$$

meaning that $x_t = \rho^t x_0 \to 0$ for initial condition x_0.

Stability of Dynamic Systems: Scalars

- If we have no initial condition, we solve the system forwards.

$$x_t = a x_{t+1}$$

for $a = 1/\rho$.

- We have a stable solution when solving forwards if

$$|a| < 1$$

such that $x_t = 0$ for all t.

Stability of Dynamic Systems: Vectors

- Now more generally when

$$\vec{x}_t = A\vec{x}_{t+1}$$

  where the above is a vector equation.

- We may have initial conditions for some variables in $\vec{x}_t$.
- We need as many stable variables in forward solution as there are missing conditions.
- E.g. simple optimal growth model:
  - Two dynamic variables (consumption and capital).
  - One initial condition for the capital stock.
  - Need consumption to be stable in forward dynamics.

Dynare and Stability

- The stability condition may not be met for all sets of parameters for a given DSGE.
- If the condition isn't satisfied in Dynare, it will spit out error

  Blanchard Kahn conditions are not satisfied: indeterminacy

- Means that there is no unique solution to the model.
- Change your parameters if this happens!

Taylor Principle

- To ensure stability of our DSGE with optimal policy implimentation, we need to adhere to the Taylor principle.
- Says that the nominal interest rate must be "sufficiently reactive" to an increase in inflation.
- Think about a policy rule of the form

$$\hat{i}_t = \hat{r}_t^n + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t^g \tag{6}$$

- Taylor principle is satisfied when $\phi_\pi > 1$, meaning that nominal rate rises by more than 1 for 1 with inflation.
- No need to worry about proving this or anything for this class. Just understand what the principle says.

Taylor Principle

- When following (6) with $\phi_\pi > 1$, $\hat{\pi}_t$ and $\hat{y}_t^g$ jump to the zero stable solution for all t.
- But this is an equilibrium outcome rather than the policy rule itself.
- Note: also other ways to achieve this.

Roadmap

Issues in Implementation

- What is the problem with this from a practical viewpoint?
- Central bank doesn't necessarily observe $\hat{r}_t^n$.
- But they're supposed to move $\hat{i}_t$ one-for-one with the natural rate!

Welfare and Simple Rules

- One can show that a simple rule of the form

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t^g$$

does pretty well when we can't set $\hat{i}_t$ with $\hat{r}_t^n$.

- That is — loss function won't be too large.

Dynare: Welfare and Simple Rules

- Consider the quadratic loss function (2) with $\omega = 1$. Use the parameter specification in exercise set 11.
- Recall that we used a value of $\phi_\pi = 1.5$.
- What happens as I keep increasing ϕ_π?
- What do the policy functions for the loss function and nominal rates for these different values?

Dynare: Welfare and Simple Rules

ϕ_π **value**	$\hat{i}_t$ **policy**
1.5	$\hat{i}_t = -0.338564\hat{a}_{t-1} + 0.010211\hat{v}_{t-1} - 0.677128\epsilon_{a,t} + 0.020421\epsilon_{v,t}$
3.0	$\hat{i}_t = -0.341767\hat{a}_{t-1} + 0.005577\hat{v}_{t-1} - 0.683534\epsilon_{a,t} + 0.011154\epsilon_{v,t}$
6.0	$\hat{i}_t = -0.343601\hat{a}_{t-1} + 0.002923\hat{v}_{t-1} - 0.687203\epsilon_{a,t} + 0.005847\epsilon_{v,t}$
100	$\hat{i}_t = -0.345495\hat{a}_{t-1} + 0.000184\hat{v}_{t-1} - 0.690990\epsilon_{a,t} + 0.000368\epsilon_{v,t}$

ϕ_π **value**	**Loss** ($\hat{W}_t$) **policy**
1.5	$\hat{W}_t = -0.147115\hat{a}_{t-1} - 0.212827\hat{v}_{t-1} - 0.294231\epsilon_{a,t} - 0.425654\epsilon_{v,t}$
3.0	$\hat{W}_t = -0.080352\hat{a}_{t-1} - 0.116243\hat{v}_{t-1} - 0.160704\epsilon_{a,t} - 0.232485\epsilon_{v,t}$
6.0	$\hat{W}_t = -0.042121\hat{a}_{t-1} - 0.060936\hat{v}_{t-1} - 0.084243\epsilon_{a,t} - 0.121871\epsilon_{v,t}$
100	$\hat{W}_t = -0.002648\hat{a}_{t-1} - 0.003830\hat{v}_{t-1} - 0.005296\epsilon_{a,t} - 0.007661\epsilon_{v,t}$

- Where the policy function for the natural rate is

$$\hat{r}_t^n = -0.345622\hat{a}_{t-1} - 0.691244\epsilon_{a,t}$$

Roadmap

Takeaways

- Optimal theoretical policy for central bank is to follow the natural rate.
- Can be approximated using a simple inflation-targeting monetary policy rule with sufficient "hawkishness".
- Higher coefficient on inflation takes us closer to the optimal rule.